

A Computational and Experimental Investigation of Multiplicity Counting with Continuous Fission Chamber Signals

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Abstract

In earlier papers we explored the possibility of extracting traditional multiplicity count rates from the continuous signals of fission chambers. It was shown that the singles, doubles and triples detection rates can be retrieved from certain one-, two- or three-point cumulants of the signals. Simulations have now been performed to investigate the impact of various parameters to the recovered detections rates. It was found that while the measurement time and the detection efficiency have a strong impact on the accuracy of estimated rates, the effect of electronic noise and parasitic pulses is either negligible or can easily be corrected for. Furthermore, while the traditional pulse counting approach underestimates the detection rates at large count rates due to dead time losses, the new method produces correct results. To demonstrate the practical use of the proposed method, measurements have been performed. The singles and doubles rates were estimated both from the moments of continuous signals as well as with pulse counting, and were compared. It was found that while the singles rates recovered from the continuous signals are slightly larger due to the α -background of the detectors, the doubles rates show a good agreement. The results show that the proposed method is a viable alternative of traditional multiplicity counting, especially at high count rates.

1 Introduction

The objective of multiplicity counting is to determine the mass of small fissile samples which is extracted from the detection rates of single, double and triple coincidences (S , D and T count rates) with pulse counting techniques [1]. The development of a method that extracts these rates from the cumulants of the continuous signals of fission chambers was the subject of earlier papers [2, 3, 4] and were presented at previous INMM conferences [5, 6], as well. In References [2] and [3] it was shown that while in fast detection systems (where the migration time of neutrons is smaller than the duration of a voltage pulse) all three detection rates can be recovered from the one point moments of the signals, in thermal systems, information on the D and T rates vanishes. This problem was resolved in [4] by extending the theory with two- and three-point moments (covariance and bicovariance function) of the signals, which allow the recovery of the D and T rates in thermal systems as well.

A computer program has been created for the efficient estimation of the cumulants of sampled detector signals and is described in Section 3. This program is used to analyze simulated

and measured detector signals as well. A computational study has been performed to investigate how various parameters affect detection rates recovered from continuous signals. In particular, the impact of the measurement time, the detection efficiency, the electronic noise, parasitic signal components and the sample emission intensity is assessed and the results are reported in Section 4.

To demonstrate the practical use of the proposed method, an experiment has been performed at the KUCA (Kyoto University Critical Assembly) facility in which neutrons from a combination of ^{252}Cf and ^{235}U samples were measured. The singles and doubles rates were determined both from the continuous signals and by pulse counting, to serve as a reference, and the applicability of the new method is assessed by comparing these two. Although results from this experiment have already been presented at an earlier INMM meeting [7], the experimental data have recently been re-evaluated to yield new, more established results; these will be summarized briefly in Section 5.

2 Theory of multiplicity counting with continuous signals

Let us assume that a measurement is performed using N detectors. Let ε_i denote the detection efficiency of the i th detector, i.e., the probability, that an emitted neutron is detected by the i th detector. Then $\varepsilon = \sum_{i=1}^N \varepsilon_i$ is the efficiency of the entire detection system, i.e., the probability, that an emitted neutron is detected by one of all the detectors.

In a traditional multiplicity counting measurement, the detection rates of the first three k -tuplets (k detected neutrons originating from the same sample emission) are determined [8]. These are called the *singles* (S), *doubles* (D) and *triples* (T) rates, respectively, and are defined as:

$$S = F \tilde{\nu}_1 \varepsilon, \quad D = F \frac{\tilde{\nu}_2}{2} \varepsilon^2 f_d, \quad T = F \frac{\tilde{\nu}_3}{6} \varepsilon^3 f_t. \quad (1)$$

Here F is the intensity of spontaneous fission in the sample, ε is the detection efficiency, $\tilde{\nu}_i$ is a modified form of the so-called Böhnel moments [9], the factorial moments of the number of emitted neutrons per one source event. The factors f_d and f_t are called the doubles and triples “gate factors” which are introduced empirically to account for the non-coincident detection of neutrons of common origin as well as to the loss of detections outside the counting window. The sought sample quantities (including the fissile mass of the sample) are then obtained from the measured values of the S , D and T rates using algebraic inversion [8].

In the newly proposed form of multiplicity counting, the continuous voltage signals of neutron detectors (primarily fission chambers) is analyzed. More concretely, various moments of the signals are determined from which the values of the S , D and T rates can be recovered. The theory was described in earlier papers in detail, therefore only a brief summary of the basic assumptions and the final results will be provided here.

The method is based on a stochastic mathematical model of the detector response to the detection of neutrons, introduced in [10]. Neutrons arrive to the detectors with a random time delay τ characterized by a density function $u(\tau)$. Each detection generates a pulse with a deterministic (constant) shape $f(t)$ (whose integral will be denoted by I) and with a random amplitude \mathbf{a} (whose mean value will be denoted by $\langle a \rangle$). It can then be shown that the mean value of the signal of detector i (κ_i), the integral of the covariance function of the signals of detector i and j ($\text{Cov}_{i,j}$) as well as the integral of the bicovariance function of the signals of

detector i , j and k ($\text{Cov}_{i,j,k}$) can be expressed as [2, 3, 4]:

$$\kappa_i = F \tilde{\nu}_1 \varepsilon_i \langle a \rangle_i I_i \quad \text{Cov}_{i,j} = F \frac{\tilde{\nu}_2}{2} \varepsilon_i \varepsilon_j \langle a \rangle_i \langle a \rangle_j I_i I_j, \quad \text{Cov}_{i,j,k} = F \frac{\tilde{\nu}_3}{6} \varepsilon_i \varepsilon_j \varepsilon_k \langle a \rangle_i \langle a \rangle_j \langle a \rangle_k I_i I_j I_k. \quad (2a)$$

By utilizing the moments of every signal, signal pairs as well as s triples, one can express the singles, doubles and triples rates with these former. In particular, by combining expressions (1) and (2), and substituting $f_d = f_t = 1$ because no detection window is applied when analysing continuous signals, we may write:

$$S = \sum_{i=1}^N \frac{\kappa_i}{\langle a \rangle_i I_i}, \quad D = \sum_{i=1}^N \frac{\kappa_i}{\langle a \rangle_i^2 I_i^2} \frac{\sum_{j=1, j \neq i}^N \kappa_j \text{Cov}_{i,j}}{N-1} + 2 \sum_{i=1}^N \sum_{j=i+1}^N \frac{\text{Cov}_{i,j}}{\langle a \rangle_i \langle a \rangle_j I_i I_j} \quad (3a)$$

and

$$T = \sum_{i=1}^N \frac{\kappa_i^2}{\langle a \rangle_i^3 I_i^3} \frac{\sum_{j=1}^N \sum_{k=1, k \neq j}^N \frac{1}{\kappa_j \kappa_k} \text{Cov}_{i,j,k}}{(N-1)(N-2)} + 3 \sum_{i=1}^N \sum_{j=1, j \neq i}^N \frac{\kappa_i \left(\frac{1}{N-2} \sum_{k=1, k \neq i, j}^N \frac{1}{\kappa_k} \text{Cov}_{i,j,k} \right)}{\langle a \rangle_i^2 I_i^2 \langle a \rangle_j I_j} \quad (3b)$$

$$+ 6 \sum_{i=1}^N \sum_{j=i+1}^N \sum_{k=j+1}^N \frac{\text{Cov}_{i,j,k}}{\langle a \rangle_i \langle a \rangle_j \langle a \rangle_k I_i I_j I_k}.$$

3 Tool for estimating the moments of recorded signals

From a data representation point of view, the voltage signal of a neutron detector recorded by some time resolution Δt forms a sequence y_1, y_2, \dots . An application named **continuous signal analyser** has been created for the (off-line) analysis of detector signals of this form. The purpose of the program is to provide an estimate for a target quantity q (which can either be the mean value or the integrals of the covariance or bicovariance functions, depending on the choice of the user) and was used to analyze both simulations (Section 4) and measurements (Section 5).

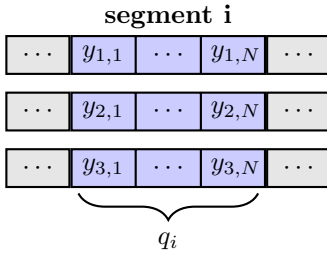


Figure 1: An illustration of dividing signal into segments during analysis.

The operation of the program is illustrated on Figure 1 and is described in the following. The signals selected for analysis are divided into segments of equals size N ; given a time resolution Δt , the length of these segments in time is $T = N \cdot \Delta t$. For each segment, a value of the target quantity, denoted by q_i , is calculated independently. Assuming that there are M segments in total, the unbiased estimator of the value of the target quantity is the sample mean $\mu_q = M^{-1} \sum_{i=1}^M q_i$; standard deviation of this estimator is $\sigma_q = \sqrt{s_q^2/M}$, where s_q^2 denotes the unbiased estimator of the sample variance and is given by $s_q^2 = (M-1)^{-1} \sum_{i=1}^M (q_i - \mu_q)^2$.

The mathematical form of q_i in the case of the above mentioned three quantities is

$$(\kappa_k)_i = \frac{1}{N} \sum_{j=1}^N y_{k,j} \quad (\text{Cov}_{1,2})_i = \frac{\Delta t}{N} \prod_{k=1}^2 \sum_{j=1}^N (y_{k,j} - \mu_k) \quad (4a)$$

$$(\text{Cov}_{1,2,3})_i = \frac{\Delta t^2}{N} \prod_{k=1}^3 \sum_{i=1}^N (y_{k,i} - \mu_k). \quad (4b)$$

The expression of $(\kappa_k)_i$ follows directly from the definition of the mean value, while the expressions of $(\text{Cov}_{1,2})_i$ and $(\text{Cov}_{1,2,3})_i$ can be obtained using the Wiener–Khinchin theorem.

4 Simulations

Although the theoretical investigations presented in papers XX have revealed some key characteristics of the new method of multiplicity counting, the effect of many important parameters is difficult to assess on a purely analytical basis. These parameters include: the time resolution by which the signal is recorded, the duration of the measurement, the presence of electronic noise in the signal, the presence of parasitic pulses (alpha or gamma background) as well as the intensity of the source. In order to assess the effect of these parameters on the estimated values of the singles, doubles and triples rates, a computational study has been performed: a large number of measurements were simulated and analyzed with the program described in Section 3, then the detection rates were obtained using formulas (3).

The values of the parameters of the simulated system – unless they are the subject of the investigation, in which case they are varied – were chosen to represent a typical multiplicity counting measurement using ^3He -gas filled detectors [1]. The sample had an intensity $10\,000\text{ s}^{-1}$ and a multiplicity, shown on Table 1, identical to the spontaneous fission multiplicity of ^{240}Pu . As in the verification step earlier, the neutrons arrived with an exponentially distributed time delay to the detectors with a mean delay of $50\ \mu\text{s}$. With three detectors, the cumulative detection efficiency was 50%. Each detection generated a pulse with shape $f(t) = c (e^{-t/\theta_{\text{pulse},1}} - e^{-t/\theta_{\text{pulse},2}})$ for $t \geq 0$ where $\theta_{\text{pulse},1} = 1\ \mu\text{s}$, $\theta_{\text{pulse},2} = 0.9\ \mu\text{s}$ (which gives a characteristic pulse length of $10\ \mu\text{s}$) and c is chosen in a way such that the amplitude of $f(t)$ is unity. The amplitude of the pulses had a gamma distribution with density function $w(a) = (\Gamma(k) \theta_{\text{amplitude}}^k)^{-1} a^{k-1} e^{-a/\theta_{\text{amplitude}}}$, where $k = 5$, $\theta_{\text{amplitude}} = 20\ \text{mV}$ (which gives a mean amplitude of $100\ \text{mV}$) and Γ denotes the gamma-function.

The simulated signals were recorded with a time resolution of $\Delta t = 0.05\ \mu\text{s}$, which large enough to properly resolve individual pulses. With one exception, the simulated measurement time was $1000\ \text{s}$ which can be regarded as a typical measurement time in multiplicity counting measurements [1]. During analysis, the signals were divided into $20\ \text{ms}$ long segments.

Table 1: Multiplicity distribution of emitted neutrons (per emission event) of the simulated sample. The values are taken from Reference [1].

n	0	1	2	3	4	5	6
$P(n)$	0.066	0.232	0.329	0.251	0.102	0.018	0.002

4.1 The impact of the measurement time

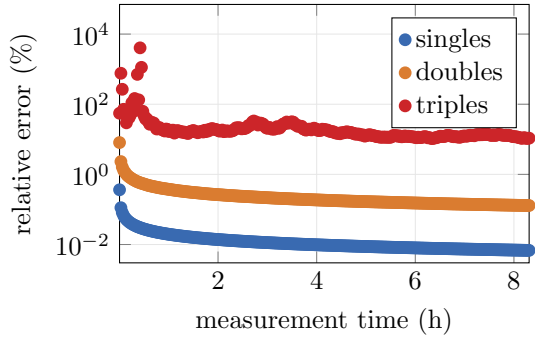


Figure 2: Relative errors of detection rates as a function of measurement time.

phenomenon was observed in all the other cases resulting in incorrect values for the triples, in the following, results will be provided only for the singles and the doubles rates. Moreover, due to the large number of cases considered, these results will be obtained from measurements lasting 1000 s (around 16 minutes).

4.2 The impact of the detection efficiency

Of all the parameters, the detection efficiency has most likely the largest influence on the performance of the assay. A traditional thermal multiplicity counter equipped with ^3He -gas filled detectors can typically provide a 40%–60% efficiency [1]; the efficiency of a system with fission chambers is one or two orders of magnitude lower. The detection efficiency in the simulations was varied in the range 1%–80%.

The results are summarized on Figure 3. One can see that the values of the estimated singles and doubles rates agree with the theoretical expectation and follow the expected linear as well as quadratic change, while the relative uncertainties in both cases decrease quickly with increasing detection efficiency.

4.3 The impact of the electronic noise

Electronic noise forming in various elements of the detection chain (e.g. in the detector or in the pre-amplifier) can easily degrade the information carried by pulses induced by detected particles [11]. To assess its effect, a Gaussian white noise has been superimposed on the simulated signals. The mean value of the noise was kept zero, while its standard deviation varied in the range 0 mV–50 mV; the upper limit corresponds to half of the mean amplitude of neutron pulses.

It was found that both the estimated values of the detection rates and their relative uncertainties were basically independent of the amplitude of the noise, which suggests that newly proposed method is, contrary to the traditional approach, is insensitive to this type of noise. This insensitivity can be explained on a theoretical basis: since the noise has a zero mean amplitude, it does not contribute to the mean value of the signal; furthermore, since the noise present in two different signals is independent (in fact, even within one signal the samples of a white noise at two distinct time instants are independent), its covariance function is zero.

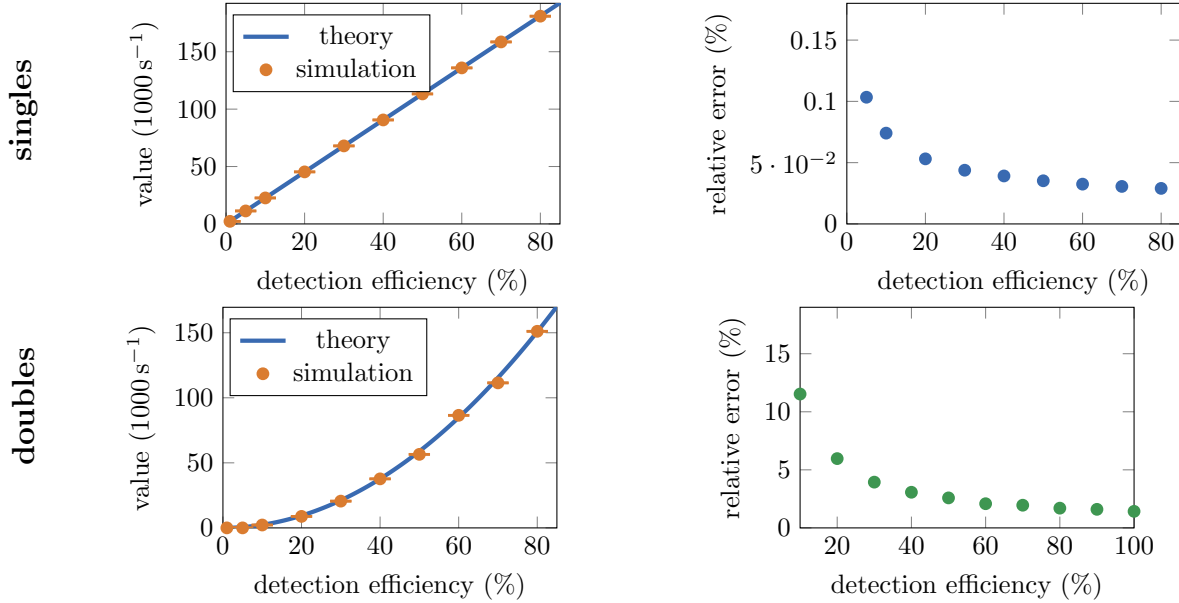


Figure 3: Values and relative errors of the singles and doubles rates as a function of the detection efficiency.

4.4 The impact of parasitic pulses

While in a traditional pulse counting mode parasitic signal components (small amplitude pulses induced by particles other than neutrons) can easily be filtered out with amplitude discrimination, they cannot be separated when analyzing continuous signals directly. To investigate this effect, a secondary particle source is considered which is independent from the neutron source and has simple Poisson statistics with detection intensity varying between 0 s^{-1} – $15\,000\text{ s}^{-1}$. Parasitic pulses have the same shape as neutron pulses but have a 10 times smaller amplitude. This simple model describes well the inherent α -background of a fission chamber or a gamma background produced by activation and fission products in ^3He -gas filled detectors [11].

The solid lines on Figure 4 show the theoretical values of the rates when no secondary source is present. One can see that the estimated singles rate increases linearly with the parasitic detection (simulation – direct), but provides correct results when the mean value of the parasitic component is subtracted from the mean value of the combined neutron and parasitic signal (simulation – direct) – a similar correction can be applied in an experiment by measuring the signal when no neutron source is present. The value of the doubles rate is insensitive to the parasitic source which is a result of two facts: first, the parasitic component in two separate signals is independent hence their cross-covariance is zero; second, the relative weight of the small amplitude parasitic pulses compared to neutron pulses is smaller in a second order moment, where the signal is raised to the second power [11]. The parasitic signal component does not seem to have an effect on the relative error of the estimated rates.

4.5 The impact of the neutron source intensity

In papers [2, 3, 4] it has been stated that one of the most appealing characteristic of the newly proposed version of multiplicity counting is the lack of dead time losses arising in the count-

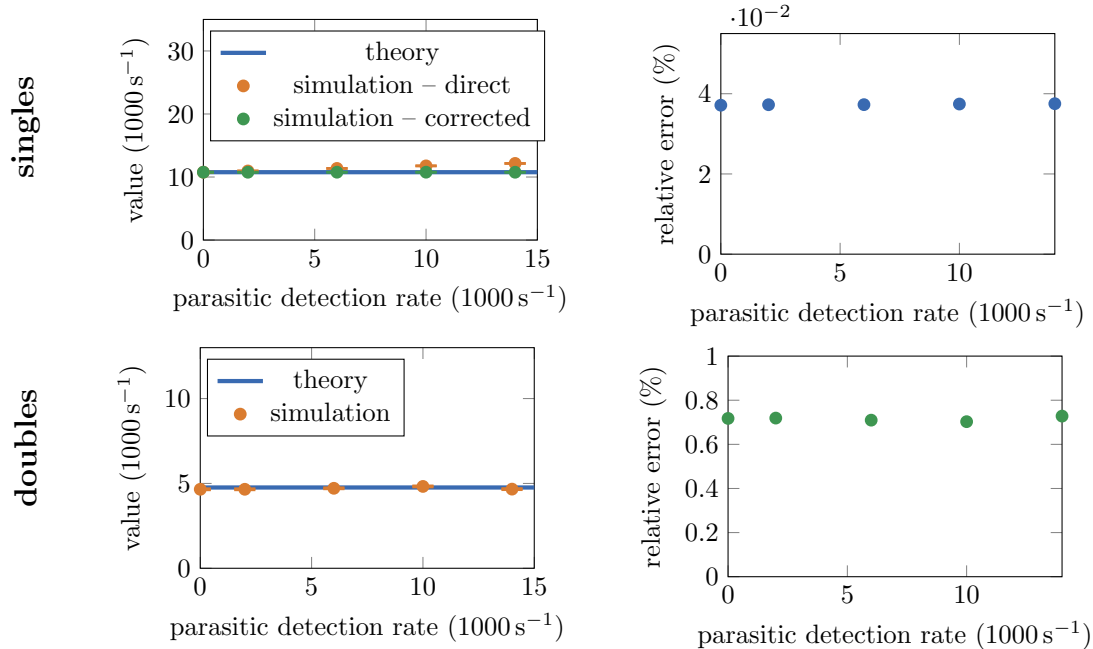


Figure 4: Values and relative errors of the singles and doubles rates as a function of the parasitic detection rate.

ing electronics when performing a traditional measurement. To illustrate this difference, the emission rates varied in the range of 1000–100000 s⁻¹ while the detection rates were estimated both with the traditional pulse counting approach (using a program that emulated an integral discriminator by counting pulses in the simulated signal with a predefined threshold level) as well as from the moments of the signal.

The results on Figure 5 show that the values obtained from the new method shown an excellent agreement with the theoretical expectations throughout the whole range of the emission rate. The agreement is also good with the values obtained from pulse counting at low emission rates, when pulses rarely overlap. However, as the emission rate increases, and pulse overlapping becomes more frequent, the traditional singles and doubles estimates underestimate the real values. It should be emphasized though that the above comparison is only conceptual and does not reflect the absolute performances of the two approaches. In practice, the dead time in the traditional method can greatly be reduced by the use of a large number of detectors and by applying dead-time correction techniques, none of which were employed in the above simulations.

5 Measurements

The geometry of the measurement set-up is shown in Figure 6. The central element is a ²⁵²Cf source with a neutron emission rate of 12930 s⁻¹. The sample is surrounded by four Westinghouse WL-8073 type *dual range fission chambers* with highly enriched uranium coating and are labeled with A–D. The system is loaded with polyethylene (PE) to thermalize fast fission neutrons. Two configurations were created in order to do measurements with two different fission rates. The first configuration contained only the ²⁵²Cf as neutron source and will be

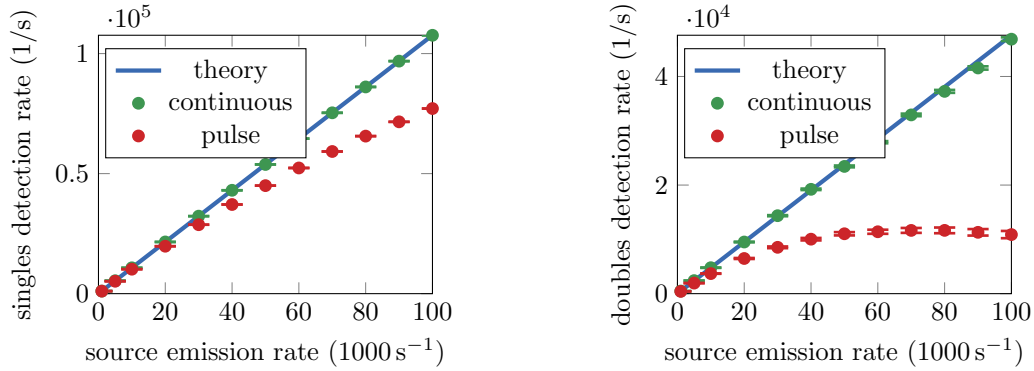


Figure 5: Values and errors of the singles and doubles rates as a function of neutron source intensity.

denoted as Cf in the following. In the second configuration 90% enriched ^{235}U plates were placed symmetrically above and below the californium to serve as a secondary neutron source (induced fission caused by the primary neutrons from the californium); this configuration will be denoted as Cf+U.

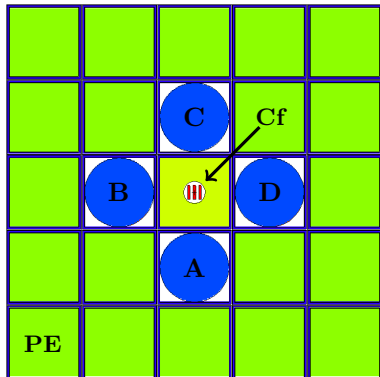


Figure 6: Horizontal view of the measurement set-up.

A 14 hour-long measurement has been performed in both configurations. The pre-amplified detector signals have been recorded with a 40 ns time resolution. The mean value of all four signals and the integrals of the covariance functions between the detector pairs A–B and C–D have been estimated by the program described in Section 3 from which the singles and doubles rates were calculated using formulas (3); the calibration factor $\langle a \rangle I$, which is the integral of the average voltage pulse, has been estimated by taking samples of individual pulses from the recorded signals using a simple program. Due to the low detection efficiency of the system, the integral of the bicovariance function of three signals could not be estimated with an acceptable statistical uncertainty, therefore no triples rates could be retrieved from this measurement.

To serve as a reference, the singles and doubles rates have been determined from the recorded times of detections using the traditional pulse counting technique described in [1]. In fact, two sets of reference values were produced: one set was obtained from the time data produced by a dedicated discriminator-counter device named NI myRIO, whereas the other came from the times determined by a simple program that scan through the recorded continuous signals and lists the times when the signal crosses a predefined threshold value. In all cases, the continuous signals have been smoothed using a simple moving average algorithm, before they were processed.

The results are summarized in Tables 2 and 3. In general, one can see that no significant difference can be observed between the values obtained in the two configurations, as it was expected from preliminary simulations.

In the case of the singles rate, the two sets of reference values (labeled as NI myRIO and program) in each case are close to each other, although the dedicated discriminator-counter device produced slightly larger values, especially in the case of detector D, whose signal was loaded with a large amount of electronic noise which caused false counts. The values obtained from the continuous signals are systematically larger compared to the reference values. This is a result of the α -background of the fission chambers, which could not be corrected for using the procedure described in Section 4.4, since no background measurement was performed.

Regarding the doubles rate, the agreement of the reference values is good for the pair A–B, but for the pair C–D, the dedicated counter produces larger values. This is again the result of the extra (false) counts caused by the noise in detector D. The doubles rates extracted from the integral of the covariance functions of the continuous signals are very close to the reference values and have smaller uncertainties.

Table 2: Estimated values of the singles rates in units of 1/s.

configuration	detector	discrete pulse counting		continuous signal analysis
		NI myRIO	program	
Cf	A	17.019 ± 0.018	17.071 ± 0.018	18.282 ± 0.704
	B	17.029 ± 0.018	17.837 ± 0.018	19.720 ± 0.838
	C	19.791 ± 0.019	19.759 ± 0.019	21.316 ± 0.920
	D	27.391 ± 0.022	21.584 ± 0.020	24.198 ± 0.777
Cf+U	A	17.370 ± 0.018	17.475 ± 0.018	18.753 ± 0.806
	B	17.627 ± 0.018	16.211 ± 0.018	18.047 ± 0.775
	C	19.949 ± 0.019	16.823 ± 0.018	18.837 ± 0.964
	D	27.440 ± 0.023	21.793 ± 0.020	24.152 ± 0.800

Table 3: Estimated values of the doubles rates in units of 1/s.

configuration	detector	discrete pulse counting		continuous signal analysis
		NI myRIO	program	
Cf	A–B	0.319 ± 0.018	0.291 ± 0.019	0.305 ± 0.006
	C–D	0.540 ± 0.027	0.451 ± 0.023	0.420 ± 0.008
Cf+U	A–B	0.304 ± 0.020	0.309 ± 0.019	0.304 ± 0.007
	C–D	0.538 ± 0.028	0.383 ± 0.022	0.376 ± 0.008

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