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Hourglass Control Convergence Studies for Hexahedral Elements Developed by the ASME Special Working Group on Computational Modeling

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ABSTRACT

The ASME Special Working Group on Computational Modeling for Explicit Dynamics was founded in August 2008 for the purpose of creating a quantitative guidance document for the development of finite element models used to analyze energy-limited events using explicit dynamics software. This document will be referenced in the ASME Code Section III, Division 3 and the next revision of NRC Regulatory Guide 7.6 as a means by which the quality of a finite element model may be judged. One portion of the document will be devoted to a series of convergence studies that demonstrates the effect of hourglass control settings on solution convergence for reduced integration elements. These convergence studies will demonstrate the importance of selecting an appropriate hourglass control setting to achieve accurate results for large deformation simulations using reduced integration elements.

In this paper, the authors present the results of a convergence study for an impulsively loaded propped cantilever beam constructed of LS-DYNA reduced integration hexahedral elements using different hourglass control settings. A large load is applied to produce large deformations and large plastic strains in the beam.

INTRODUCTION

Explicit finite element codes, such as LS-DYNA [1] and ABAQUS [2], are used to analyze storage casks and transportation packages for energy-limited events, such as drop impact, puncture and aircraft crash. These codes have evolved to become sufficiently sophisticated and robust as to be able to predict the response to such events with reasonable accuracy. Such results are only achievable, however, by analysts who possess intimate knowledge of structural behavior and an understanding of how to properly construct a finite element model to produce accurate results using these codes.

In the hierarchy of complexity for finite element structural analysis is explicit dynamic analysis, which is typically used to solve crash and impact problems, followed by implicit dynamic analysis, which is typically used to solve problems in forced vibrations and ground motion, and finally static analysis.

Engineers who first encounter explicit dynamics codes, and who may be well versed in static analysis and implicit dynamics, soon become aware of the new challenges presented by crash and impact problems using explicit dynamics codes. To help address these new challenges, the ASME Special Working Group (SWG) on Computational Modeling for Explicit Dynamics was formed in August 2008. The purpose of the SWG is to create a quantitative guidance document for the development of finite element models used to analyze energy-limited events using explicit dynamics codes. This guidance document will support the use of strain-based acceptance criteria being developed for Section III, Division 3, WB/WC 3700 [3]. This document will become a Non-Mandatory Appendix in the ASME Code Section III, Division 3 [4] and be referenced in the next revision of NRC Regulatory Guide 7.6 [5] as a means by which the quality of a finite element model may be judged.

Of all the considerations that go into constructing an accurate finite element model, the choice of element type and level of refinement of the element mesh are the most fundamental. Therefore, the SWG's first undertaking was to develop a series of element convergence studies that can aid designers in establishing the mesh refinement requirements necessary to achieve accurate results for a variety of different elements types in regions of high plastic strain. These mesh convergence studies will be incorporated into the guidance document for use.

The convergence studies include single point (reduced) integration type elements. While mesh refinement is fundamental to obtaining converged results, the single point integration element also needs to have the appropriate hourglass control setting to obtain accurate results. Hourglass control is needed because reduced integration elements can deform in non-physical modes other than rigid body modes due to the use of single point Gauss quadrature. These "mathematical" modes are historically called hourglass modes because they deform elements into an hourglass shape (i.e., zigzag shape). This type of deformation is now commonly referred to as a zero-energy deformation mode and can propagate in a mesh and invalidate results if not controlled. The occurrence of hourglass modes in an analysis should always be minimized so as not to significantly influence results. Explicit codes apply hourglass control to inhibit the formation of hourglass modes.

Hourglass control falls into two basic categories: stiffness based control and viscous based damping control. The type of hourglass control to employ typically depends on the loading condition; under some loading conditions stiffness control is more effective while for others viscous damping control is more effective. Hourglass control works by artificially stiffening the elements to inhibit the formation of the hourglass modes. However, this has the side effect of introducing artificial energy into the deformation response of the elements. This artificial energy (referred to as hourglass energy) needs to be large enough to control hourglassing but small enough so as not to overly stiffen the response of the elements.

A way to measure the acceptability of the hourglass energy introduced into a solution is to compare it to the component's internal energy. Generally, hourglass energy is considered acceptable if it is kept relatively small with respect to the peak internal energy of the component.

In this paper, the authors present the convergence study results for a propped cantilever beam constructed of reduced integration hexahedral elements using LS-DYNA. The FEA model will use both viscous and stiffness based hourglass control settings to assess effect on results. The beam is loaded by a uniformly distributed load that is ramped up over a finite time to a constant value. The beam problem is illustrated in Figure 1.

BEAM PROBLEM DEFINITION

The propped cantilever beam is 20 inches long with a 1-inch square cross section. The beam is built-in on the left end (ux=0) and simply supported at the two bottom corners (uy=0) as shown in Figure 1. The beam is analysed as a plane-strain problem with only one element through the width (z-directon). Therefore, all nodes in the model are restrained from displacement in the z-direction (uz=0).

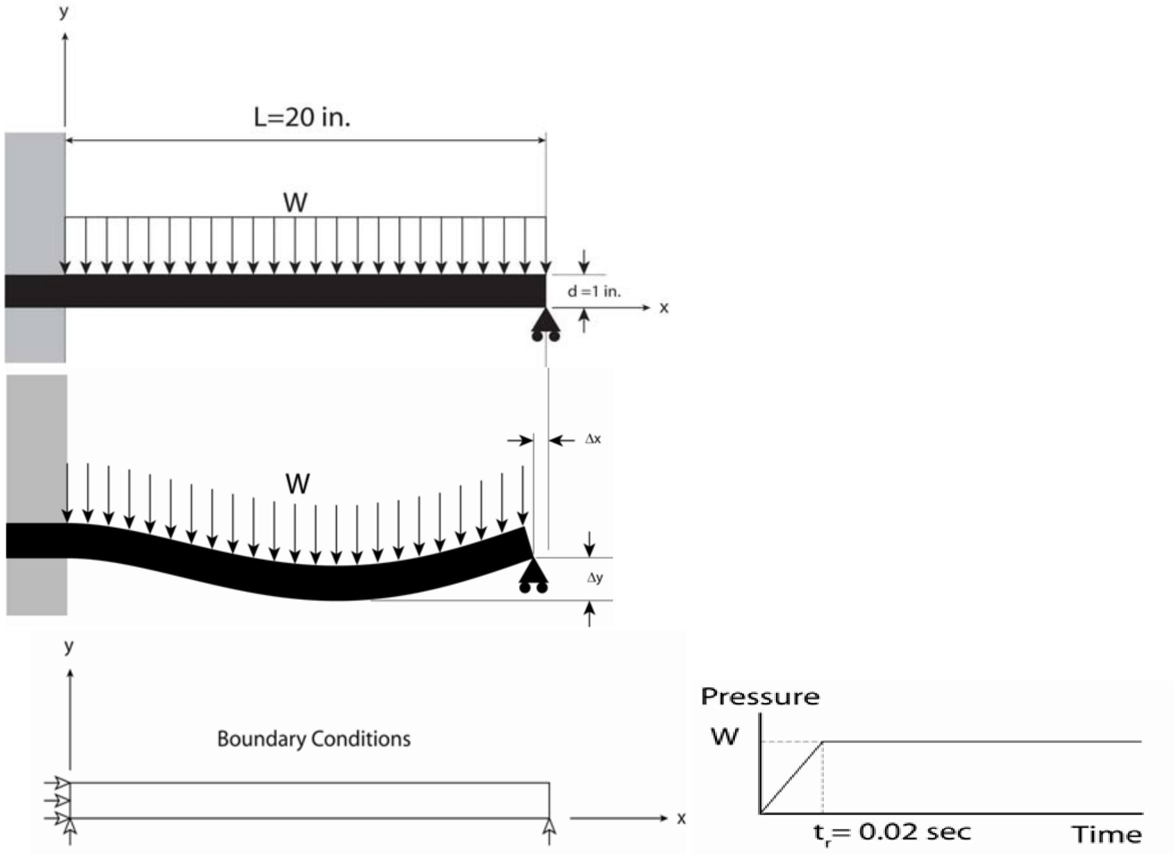

Figure 1: Propped cantilever convergence problem

The beam is assumed to be a stainless steel material with a yield strength of 30 ksi. The material model is a power-law hardening model: $\sigma = \sigma_y + A \cdot \varepsilon_p^{\,n}$, with σ_y = 30 ksi, strength coefficient A = 192 ksi, and hardening exponent n = 0.74819. The material has an elastic modulus E = 28e6 psi, a poisson's ratio ν = 0.3, and a mass density of ρ = 7.39e-4 lbm/in^3.

The loading is a downward uniformly distributed load of magnitude W applied on the top surface of the beam as shown in Figure 1. The pressure load ramps from zero to value "W" over a time interval of t_r = 0.02 seconds, and then remains constant. The pressure loading is defined to remain vertically oriented throughout the beam deflection. For this problem, a pressure load of 500 psi is used to induce large plastic deformation.

The beam is meshed with reduced integration hexahedral elements. The problem is evaluated using mesh densities of 5, 7 and 9 elements through the beam thickness using an element aspect ratio of 1. This element aspect ratio results in beam mesh densities of 5x100, 7x140, and 9x180.

Solution convergence will be assessed using both viscous and stiffness based hourglass control with varying hourglass control coefficients. Results will be obtained after the full load has been applied and the beam has come to rest. To accomplish this, damping can be applied well after full load application. The following outputs will be obtained:

- maximum y-deflection of the beam (to assess solution convergence) and

- ratio of hourglass energy to internal energy (referred to as the hourglass energy ratio).

MODEL SETTINGS AND ASSUMPTIONS

The problem was analyzed using LS-DYNA with the following modeling settings and assumptions:

- The power-law hardening material is modeled using *MAT_SIMPLIFIED_JOHNSON_COOK, which includes the power law relationship ($a + b \cdot \varepsilon^n$).
- The load is applied using distributed nodal load, calculated from the pressure load on the initial area of the top surface of the beam.
- Mass damping is applied well after full load application to damp out oscillations.

The hourglass control settings shown in Table 1 are evaluated. Type 1 is viscous based and is the default setting used in LS-DYNA. Viscous hourglass control is recommended for problems deforming with high velocities. Type 6 is stiffness based and is available only for solid elements. Stiffness hourglass control is often preferable for lower deformation velocities, especially if the number of time steps is large.

Table 1: Hourglass Control Settings used in Study

Hourglass Control	LS-DYNA	
	Type (IHQ)	Coefficient (QH)
Viscous	1	0.1
		0.2
Stiffness	6	0.02
		0.10
		0.15
		0.20

RESULTS

Viscous Hourglass Control (IHQ=1)

The results with viscous hourglass control were not acceptable. For all solutions, the elements started hourglassing almost immediately at the two support ends. The hourglassing then propagated towards the center of the beam. The final displacement result for the 5x100 beam is shown in Figure 2 and Figure 3 for the hourglass coefficient QH=0.1; results were similar for QH=0.2. The displacement results and hourglassing were similar for the 7x140 and 9x180 beams. Clearly, viscous hourglass control was not effective in preventing element hourglass modes.

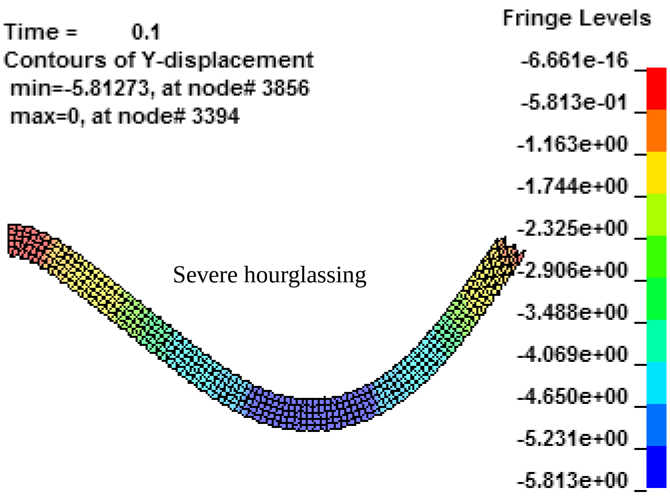

Figure 2: 5x100 Mesh, Final Deformation, Viscous Hourglass Control (IHQ=1, QH=0.1)

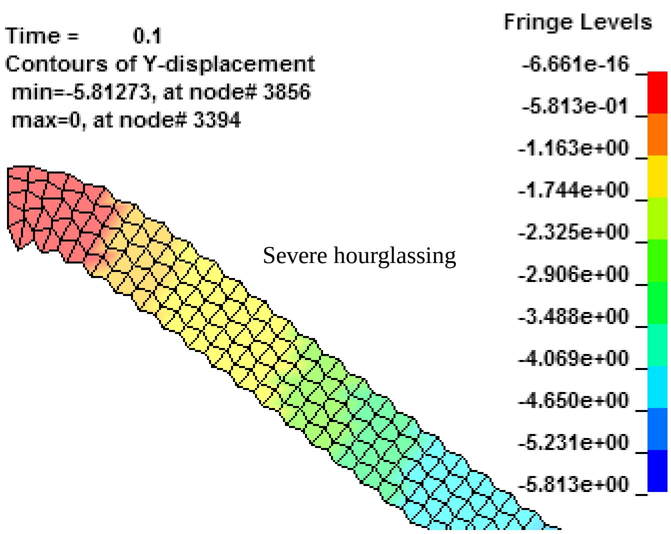

Figure 3: 5x100 Mesh, Final Deformation, Viscous Hourglass Control (IHQ=1, QH=0.1)

Stiffness Hourglass Control (IHQ=6)

The stiffness hourglass control was effective in all cases in preventing hourglass modes. No hourglassing was observed in any of the beam elements. The final displacement result for the 5x100 beam is shown in Figure 4 and Figure 5 and for the hourglass coefficient QH=0.02. Figure 6 shows the final displacement of the built in end of the 9x180 beam for hourglass coefficient QH=0.02.

The final results for hourglass energy ratio (hourglass energy/internal energy) and maximum y displacement are summarized in Table 2 and plotted in Figure 7. The results show that increasing the hourglass coefficient causes an increase in the hourglass ratio and a reduction in maximum beam displacement. This is to be expected because increasing the hourglass coefficient increases the artificial stiffness added to the elements to inhibit the formation of hourglass modes. For this problem, the results show that the stiffness based hourglass control coefficient can have a significant effect on results.

After reviewing the results, it was decided to evaluate two additional cases for the 9x180 beam. In one case, an extremely low hourglass coefficient was used (QH=0.002) to assess effect on results. In the other case, the element formulation was changed to full selectively-reduced integration to obtain a solution that was free of the influence of hourglass control. The final results are summarized in Table 2. Figure 8 shows the final displacement of the built in end of the 9x180 beam for the hourglass coefficient QH=0.002. Some hourglassing is observed in the elements. The maximum y displacement for the 9x180 beam using full selectively-reduced integration elements -5.39 inches. This displacement can be used to gage the appropriate level of hourglass control applied to the reduced integration elements.

CONCLUSIONS

From the results presented herein, the following conclusions and observations are drawn for the viscous (IHQ=1) and stiffness (IHQ=6) based hourglass control:

- Viscous hourglass control was not effective in preventing hourglassing of the beam elements. Viscous hourglass control is recommended for problems deforming with high velocities. The results for this problem support this recommendation.
- Stiffness hourglass control was effective in preventing hourglassing. Stiffness hourglass control is often preferable for lower deformation velocities, especially if the number of time steps is large. The results for this problem support this guidance.
- Stiffness hourglass control can have a significant effect on results. For this problem, the effect on displacement appears to be linear and is more exaggerated for the cruder mesh densities (see Figure 7).
- Using QH=0.02 for the three different mesh densities results in maximum displacements that are fairly close to one another. The results are close the maximum displacement results using the 9x180 mesh with full selectively-reduced integration elements.
- The hourglass energy ratio is typically used to gage the acceptable level of artificial stiffness added to the elements in a solution. The recommended limiting value for this ratio was found to vary in a literature search (1% - 15%). A value of less than 10 percent was found to be commonly used as a "rule-of-thumb", but with an expectation that sensitivity runs be performed to assess acceptability. For this problem, a value of less than 10 percent would not necessarily provide a converged solution (see Figure 7). Observing the results, it appears that an approximately converged solution could be indicated when the different mesh density results do not vary significantly when using that same hourglass control coefficient. When using QH=0.02, the maximum displacement results are close to one another and to the final displacement of the 9x180 beam meshed with full selectively-reduced integration elements. At higher QH values, note that there is significant spread between final displacement results.
- The displacement results should always be reviewed to look for hourglass modes. The occurrence of hourglass modes in an analysis can influence results. For the case where the hourglass coefficient was set extremely low (QH=0.002), some hourglassing was present in the beam elements (Figure 8) and caused an increase in maximum displacement.

REFERENCES

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3. Snow S. D., Morton D. K., Pleins E. L., Keating R., “Strain-Based Acceptance Criteria for Energy-Limited Events,” PVP2009-77122, ASME PVP Conference, July 26-30, 2009, Prague, Czech Republic.
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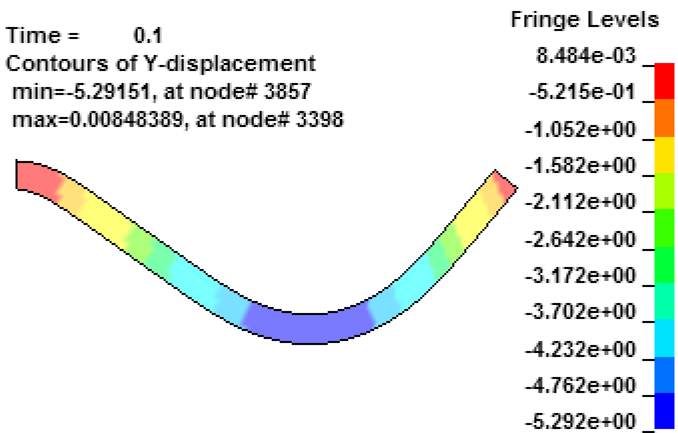

Figure 4: 5x100 Mesh, Final Deformation, Stiffness Hourglass Control (IHQ=6, QH=0.02)

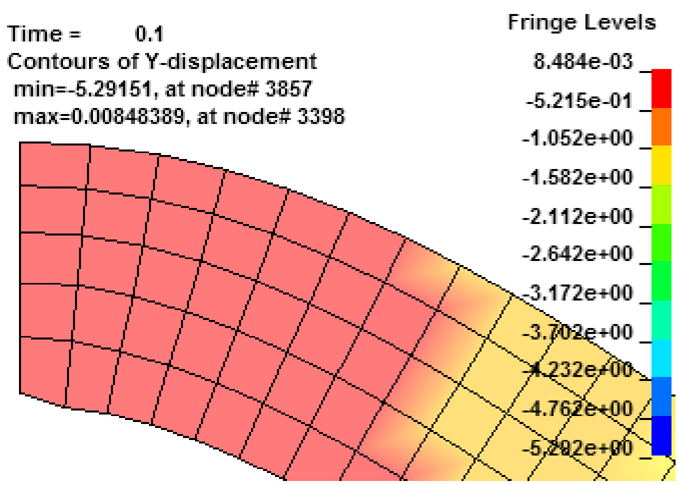

Figure 5: 5x100 Mesh, Final Deformation, Stiffness Hourglass Control (IHQ=6, QH=0.02)

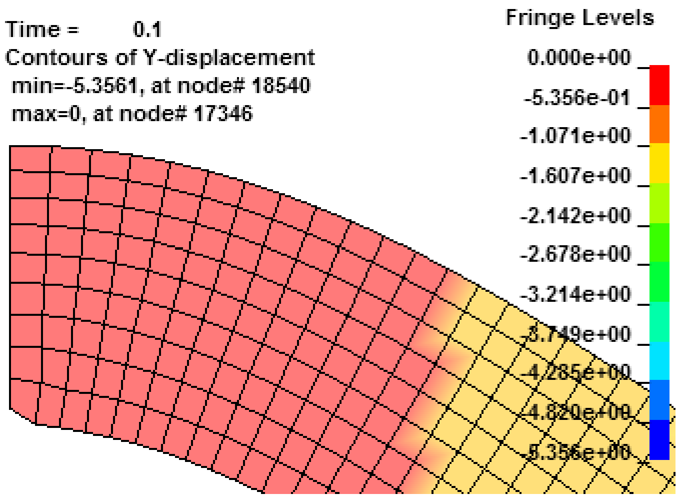

Figure 6: 9x180 Mesh, Final Deformation, Stiffness Hourglass Control (IHQ=6, QH=0.02)

Table 2: Stiffness Hourglass Control Results (IHQ=6)

Beam	HGlass Coeff (QH)	HGlass Energy Ratio(1) (%)	Ymax(2) (in.)	ΔYmax(3) (%)	HGlassing Observed
5x100	0.02	2.00	-5.29	-	no
	0.10	7.68	-4.44	16.07	no
	0.15	10.4	-4.03	23.44	no
	0.20	12.7	-3.67	30.62	no
7x140	0.02	1.13	-5.32	-	no
	0.10	4.31	-4.83	9.21	no
	0.15	6.02	-4.57	14.10	no
	0.20	7.56	-4.33	18.61	no
9x180	0.02	0.81	-5.36	-	no
	0.10	2.78	-5.03	6.16	no
	0.15	3.91	-4.85	9.51	no
	0.20	4.98	-4.69	12.50	no
Additional Cases					
9x180	0.002	0.49	-5.52	-	yes
9x180 Full SR(4)	n/a	n/a	-5.39	-	n/a

(1) 100 x (Hourglass Energy/Internal Energy)
(2) Maximum beam deflection after beam has come to rest
(3) Percent difference with Ymax at hourglass coefficient 0.02.
(4) Full selectively-reduced integration elements do not require hourglass controls.

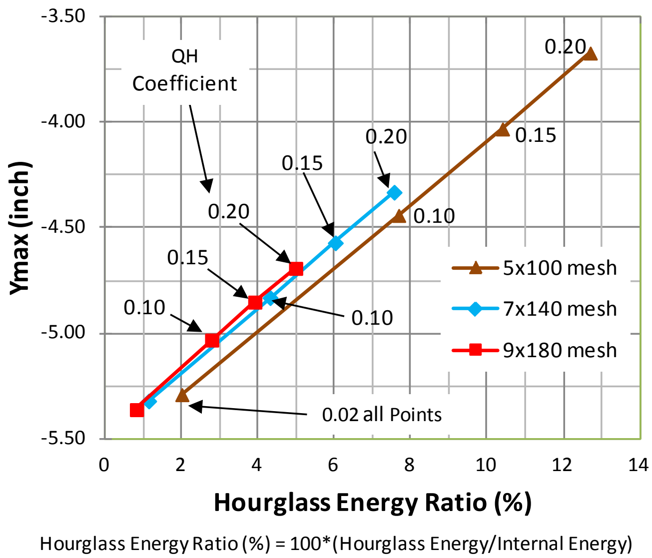

Figure 7: Stiffness Hourglass Control (IHQ=6), Ymax vs. Hourglass Energy Ratio

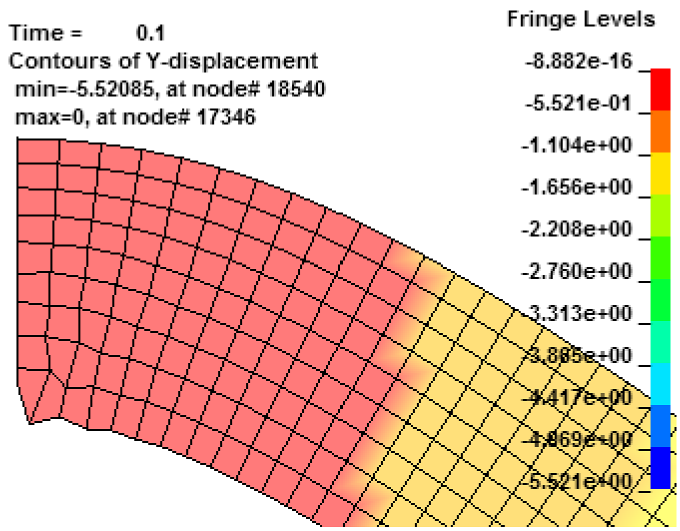

Figure 8: 9x180 Mesh, Final Deformation, Stiffness Hourglass Control (IHQ=6, QH=0.002)