# Analytical, Numerical and Experimental Investigations on the Impact Behaviour of Packagings for the Transport of Radioactive Material under Slap Down Conditions

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**ABSRTACT**

This paper decribes a methodical way to find critical drop angles or better a range of drop angles for oblique drops of a packages used for the transport of radioactive materials.

## INTRODOCTION

Concerning approval design tests the IAEA regulations for the safe transport of radioactive materials specify 9 m drop tests onto an unyielding target to evaluate the packaging response to mechanical tests demonstrating the safety under accident conditions. The orientation of the packaging, i. e. point and angle of impact in the drop test must be chosen in a manner that maximum damage occurs with regard to the safety criteria. The safety criteria are in particular the leak tightness of the lid closure system, the integrity of the containment components (body, lids, lid screws) and the subcriticality of the fissile contents. For most packages the worst case is not a single event, represented by one drop test. The worst case for the safety criteria integrity of the container body must not be automatically the worst case for the criteria of leak tightness, etc. For this reason most package drop tests may consist of a series of tests at various orientations so that every safety relevant components suffers maximum damage. Possible orientations are the horizontal, the vertical, the corner and the oblique drop.

The oblique drop, subject of this paper, do not impact the target with the container centre of gravity directly above the point of impact like in a corner drop, so that after a primary impact of one container head, the container is set into rotation. This causes a second impact onto the other end of the container with an impact velocity possibly much higher than the velocity reached from the free drop of 9 meters.

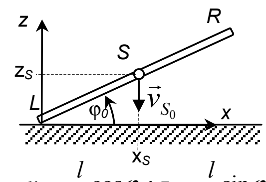
In order to evaluate on the different safety criteria, one of the difficulties is to evaluate the effects of slap down impacts depending on the chosen angle. To solve this problem, BAM had undertaken an analytical analysis of the slap down kinematics. We assumed that the package behaves like a rigid body, and looked at four borderline cases of impact conditions, an ideal elastic or plastic impact, with friction (perfectly rough impact) or without friction (perfectly smooth impact) between container and target during primary impact. In two cases we didn't find closed analytical formulas but got numerical solutions using the software program MATHEMATICA. The derivation of our solutions for the different borderline cases were discussed in detail in the next chapter. After that we will present our finite element calculations and some experimental results with the aim to check our analytical solutions. Based on these analysis we are able now to define much more precisely worse case drop angle which should be used to get high structure loading in a real drop test or in a numerical three-dimensional drop simulation.

## ANALYTICAL MODEL

The analytical model describes the impact of an uniform rigid rod of length *l*, mass *m*, and moment of inertia about the mass centre S of $\theta_S = (m\, l^2)/12$ on a rigid, horizontal plane, as shown in *Figure 1.*

The x-axis is chosen tangential, the z-axis normal to the contact surface in the contact point L. It is presupposed, that the model copies in a good estimation the rigid body characteristic of a real container (see *Experimental Results*).

The rod impacts at first the rigid target with its left end L under the impact angle $\varphi_0$ with the velocity $\vec{v}_{S_0} = \{\dot{x}_{S_0}, \dot{z}_{S_0}\}$ . After this impact the mass centre S has the final linear velocity $\vec{v}_{S_2} = \{\dot{x}_{S_2}, \dot{z}_{S_2}\}$ and the final angular velocity $\omega_2$. Further the rod executes a plane motion in the gravity field, described with the velocity of mass center S $\vec{v}_S(t) = \{\dot{x}_S, \dot{z}_S\}$ and the angular velocity $\omega(t)$ about S since it impacts a second time with its right end R. This second impact the so-called slap-down impact.




$x_S = \frac{l}{2}\cos\varphi_0 ;\, z_S = \frac{l}{2}\sin\varphi_0$

*Fig.1.* Impact of a rigid bar onto a rigid horizontal plane

During the primary impact, at the time $t = t_0$ the principle of linear and angular momentum provides the relations

$$I_x = m(\dot{x}_{S_2} - \dot{x}_{S_0}) \tag{1}$$

$$I_z = m(\dot{z}_{S_2} - \dot{z}_{S_0}) \tag{2}$$

$$\theta_S(\omega_2 - \omega_0) = I_z\, x_S - I_x\, z_S = I_z \frac{l}{2}\cos\varphi_0 - I_x \frac{l}{2}\sin\varphi_0 \tag{3}$$

where $I_x$ is the normal and $I_z$ the tangential impulse, produced by the collision ([1], [2]). The initial conditions for an IAEA 9 m drop at time $t = t_0$ are

$$\omega_0 = 0\, , \dot{x}_{S_0} = 0\, , \;\; \dot{z}_{S_0} = -v_0 \;\; ,$$

where $v_0$ is the initial impact velocity and $\omega_0$ the initial angular velocity.

The velocity $\vec{v}_L = \{\dot{x}_L, \dot{z}_L\}$ of the rod's left end L is given in general form, with $\omega = -\dot{\varphi}$ by the equations

$$\dot{x}_L = \dot{x}_S - \omega\, z_S = \dot{x}_S - \omega \frac{l}{2}\sin\varphi\, ; \;\; \dot{z}_L = \dot{z}_S + \omega\, x_S = \dot{z}_S + \omega \frac{l}{2}\cos\varphi\, . \tag{4}$$

Using the coefficient of restitution *k* as defined in [3], as ratio of final to initial normal velocity in point L, this component of the velocity after the first impact can be expressed by the formula

$$\dot{z}_{L_2} = -k\, \dot{z}_{L_0}\, , \tag{5}$$

where $\dot{z}_{L_0}$ is the z-component of the velocity of point L before the first impact and $\dot{z}_{L_2}$ after the first impact, at time $t = t_2$. The coefficient of restitution *k* describes the degree of plasticity of the collision. The impact is perfectly plastic for $k = 0$, partially elastic for $0 < k < 1$ and perfectly elastic for $k = 1$.

The normal velocity of mass centre $\dot{z}_{S_2}$ at time $t = t_2$ , using equation (4) and (5) is now given by

$$\dot{z}_{S_2} = k\, v_0 - \omega_2 \frac{l}{2}\cos\varphi_0\, .$$

If the impact is frictionless -perfectly smooth [1]- the impulse has only a z - component $I_z$, the horizontal component is zero

$$I_x = 0$$

so that no change in the horizontal velocity of the centre mass occurs

$$\dot{x}_{S_2} = \dot{x}_{S_0} = 0$$

what means that it moves only in vertical direction, as shown in *Fig 2*.

If the impact is perfectly rough [1] (*Fig.3*), the impulse consists of a z - component $I_z$ and a horizontal $I_x$ , no motion in horizontal direction can occur for point L during and after impact:

$$\dot{x}_{L_2} = 0 \Rightarrow (4) \Rightarrow \dot{x}_{S_2} = \omega_2 \frac{l}{2} \sin\varphi_0 .$$

The final angular velocity $\omega_2$ and linear velocity $\vec{v}_{S_2}$ of mass centre S solving the equations (1) to (5) are summarised in *Table 1*.

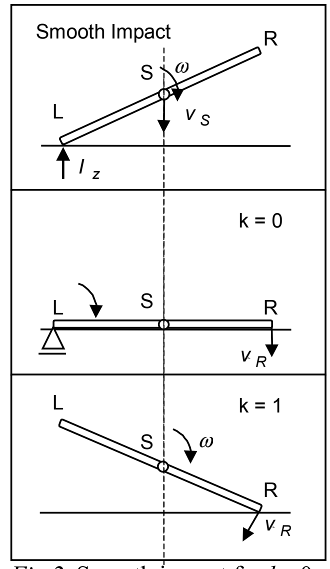


*Fig.2.* Smooth impact for *k* =0 and *k* = 1.

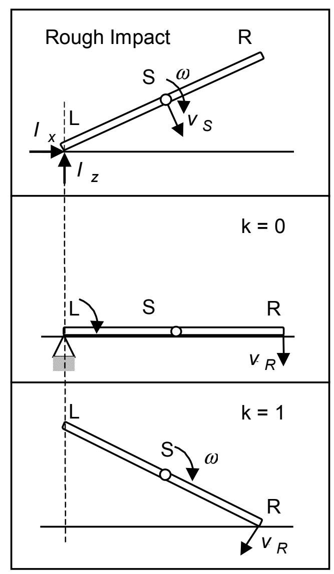


*Fig.3.* Rough impact for *k* =0 and *k* = 1.

| | Perfectly Rough Impact | Perfectly Smooth Impact |
|---|---|---|
| $\omega_2$ | $\frac{3(1+k)}{2l} v_0 \cos\varphi_0$ | $\frac{6(1+k)}{l(3\cos^2\varphi_0+1)} v_0 \cos\varphi_0$ |
| $\dot{x}_{S2}$ | $\frac{3(1+k)}{4} v_0 \cos\varphi_0 \sin\varphi_0$ | 0 |
| $\dot{z}_{S2}$ | $[k - \frac{3(1+k)}{4} \cos^2\varphi_0] v_0$ | $\frac{(3\cos^2\varphi_0 - k)}{(3\cos^2\varphi_0+1)} v_0$ |

*Table 1.* Final angular and linear velocities after first impact.

After the first impact the motion of the rod can be described by a translatory motion of the mass centre while rotating about its centre of mass in the field of gravity. The gravity force is the only working outer force during executing a plane motion. In the case of rebounding of the rod end L (restitution coefficient k > 0), the time at which second impact (slap down) occurs is defined by the condition for the z – co-ordinate of the right end R with

$$z_R(\tau_*) = \frac{l}{2}\cos\varphi_0 + \dot{z}_{S_2}\tau_* - \frac{g\tau_*^2}{2} + \frac{l}{2}\sin(\varphi_0 - \omega_2\tau_*) = 0 \qquad (6)$$

where $\tau_* = t_* - t_2$ is the time period between first and second impact and $z_R$ is the z – co-ordinate of the right end R. Equation (6) was solved numerical using [3]. The final linear slap down velocity of the right rod end R $\vec{v}_{R_*}(t_*) = \left\{\dot{x}_{R_*}, \dot{z}_{R_*}\right\}$, the mass centre S $\vec{v}_{S_*}(t_*) = \left\{\dot{x}_{S_*}, \dot{z}_{S_*}\right\}$ and the final angular velocity $\omega_*$ are summarised in *Table 2*.

In the case of no rebound ($k$ = 0) the angular velocity can be calculated directly with the law of conservation of energy after e.g. [2].

| | $k > 0$ | $k = 0$ | |
|---|---|---|---|
| | Rough/ Smooth Impact | Rough Impact | Smooth Impact |
| $\omega_*$ | $\omega_2$ | $\sqrt{\omega_2^2 + \frac{3g}{l}\sin\varphi_0}$ | $\frac{1}{2}\sqrt{(3\cos^2\varphi_0 + 1)\omega_2^2 + \frac{12g}{l}\sin\varphi_0}$ |
| $\dot{x}_{S*}$ | $\dot{x}_{S2}$ | 0 | 0 |
| $\dot{z}_{S*}$ | $\dot{z}_{S2} - g\tau_*$ | $-\omega_* \frac{l}{2}$ | $-\omega_* \frac{l}{2}$ |
| $\dot{x}_{R*}$ | $\dot{x}_{S*} + \omega_* \frac{l}{2}\sin(\varphi_0 - \omega_*\tau_*)$ | 0 | 0 |
| $\dot{z}_{R*}$ | $\dot{z}_{S*} - \omega_* \frac{l}{2}\cos(\varphi_0 - \omega_*\tau_*)$ | $-\omega_* l$ | $-\omega_* l$ |

*Table 2.* Equations governing the linear and angular velocity at time $t_*$ of the Slap-Down Impact.

## RESULTS FROM ANALYTICAL CALCULATION

The equations governing the linear and angular velocity of the slap-down impact end R, were evaluated for the borderlines perfectly smooth impact and perfectly rough impact each with $k = 0$ and $k = 1$ (see *Table 1*). The numerical calculation was carried out with an initial velocity $v_0$ of 13.3 m/s resulting from a 9 m drop and a rod length $l$ of 4750 mm varying the impact angle.

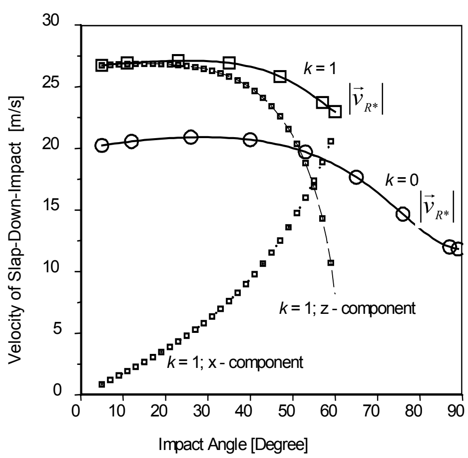



*Fig.4.* Perfectly smooth impact. Calculated velocity components and magnitude of the slap down impact for a rod with length 4750 mm.

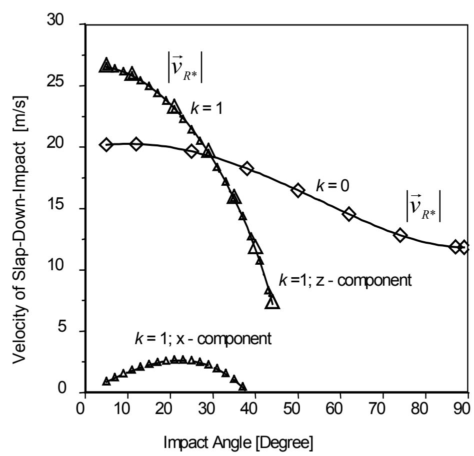



*Fig.5.* Perfectly rough Impact. Calculated velocity components and magnitude of the slap down impact for a rod with length 4750 mm.

The length is related to a cask for transport of fresh fuels, ANF-10, with that BAM had carried out a 9 m drop test with an impact angle of 15 degree [5]. The variation of lengths between 2 m and 6 m didn't show worth mentioning differences in kinematic results, so that the presented results for $l$ = 4750 mm are representative for the mentioned range.

*Figures 4* and *5* show the magnitude $\left|\vec{v}_{R*}(t_*)\right|$ (in the following text $v_{R*}$) the horizontal component $\dot{x}_{R*}$ and vertical component $\dot{z}_{R*}$ of the slap down velocity of end R for smooth and rough impact depending on impact angle. From reason of presentation, the velocity components in the *Figures 4* and *5* are shown as absolute values. But the direction can easily be seen in *Figures 2* and *3*.

The case of a smooth and perfectly elastic impact causes naturally a much higher $v_{R*}$ than a perfectly plastic impact (*Fig. 4*) and for both cases a significantly higher velocity than the initial velocity 13,3 m/s resulting from the 9 m drop height. In a wide range between 5° and 45° $v_{R*}$ for $k$ = 0 and $k$ = 1 isn't much changing.

The rough impact (*Fig.5*) shows a relative sharp decline of $v_{R*}$ for $k$ = 1 and an increasing impact angle. For $k$ = 0 $v_{R*}$ has up to 25° only a slight decrease in magnitude and then for angles greater 25° the decreasing gets significant.

Regarding the results of the four borderlines, the comparison between the velocities (*Fig.7*) shows, that

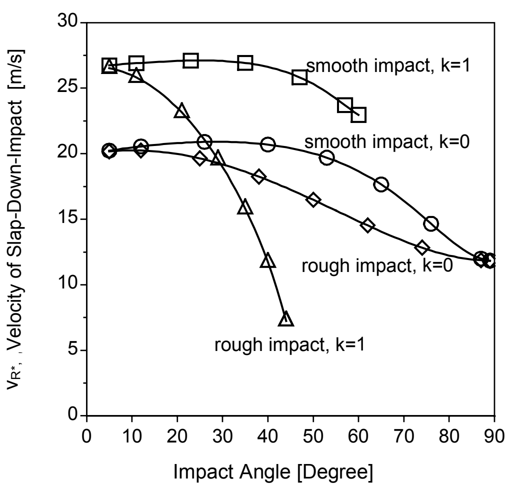



*Fig.7.* Comparison between smooth and rough impact. Magnitudes of the slap-down velocities.

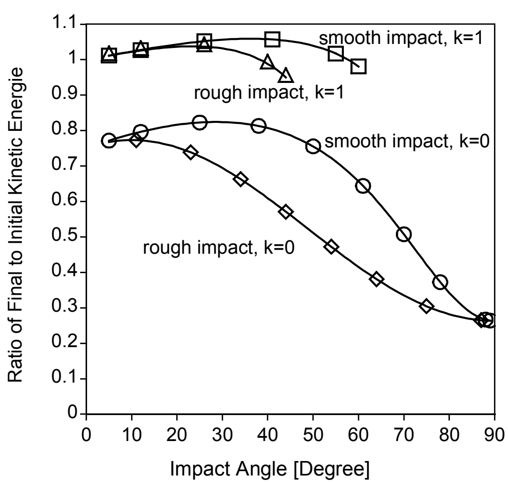



*Fig.8.* Comparison between smooth and rough impact. Ratio of final to initial kinetic energy.

the perfectly elastic, smooth impact yields the highest impact velocity. The maximum velocity isn't much changing in a wide band of impact angle except the rough impact with $k$ = 1.

*Figure 8* shows the ratio of final to initial kinetic energy depending from impact angle. In the case of impacts with $k$ = 1 and impact angles up to nearly 40 degree, the ratio is in a range between 1 and 1.05. The reason is, that the second impact has additional energy from the rotation of mass center S from its elevated position. Also we see, that as well in a perfectly plastic impact ($k$ = 0) and impact angles up to 30 degree the kinetic energy remained for slap down is 70 % - 80 % of the initial kinetic energy.

## FINITE ELEMENT CALCULATION

The finite element calculation was used to check our analytical models and for further investigations in the structure dynamics of slap down impacts (see [5]). The calculations by varying the impact angle were carried out with ABAQUS/EXPLICIT [6].

Corresponding to the analytical model the rod in the FE calculation was defined as RIGID BODY [6] (modelled by HEX8 elements) with a length of 4750 mm. The target was modelled as rigid. Due to the rigid body definition only the perfect elastic smooth and rough impacts could be simulated directly. The results show a very good conformity with those obtained from the analytical model. *Figure 9* shows for example the slap down velocities for the smooth impact in comparison between FE calculation and analytical calculation. The small difference between the curves is caused by the cross section of 10 mm x 10 mm used for the rod in the FE calculation. A cross section going to zero would match the thin rod in the analytical model and would cause in two identical curves.

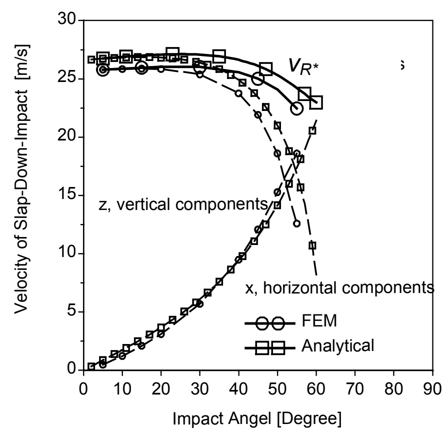



*Fig.9.* Comparison of analytical results with results obtained from FE calculation for perfectly elastic smooth impact.

Other cross sections used in the FE calculation like for ex. 500 mm x 600 mm, according to the outer dimensions of the container ANF-10 [7] showed little differences in results up to 40 degree impact angle. Beyond 40 degree the decrease is higher.

## COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL RESULTS

The experimental data to compare with calculations is obtained from drop tests with various casks onto a rigid target from a height of 9 m. The impact angle in each drop was 15°. The casks considered have lengths between 4500 mm and 5500 mm and masses between 315 kg and 20950 kg. The cross section dimensions are small in relation to their length.

*Figure* 10 e.g. shows a CASTOR VHLW equipped with shock absorbers after the 9 m, declined drop test. The shock absorber of the one end which hits first is less damaged then the opposite end slapped down on the impact target.

*Fig.10.* A CASTOR VHLW cask after the 15° declined, 9 m drop test onto a rigid target. In the foreground the higher damaged shock absorber caused by slap-down impact.

The other drop test we compared were performed with different types of new package designs for the transport of fresh fuel called ESBB, ANF-10 and ANF-18. The design of the packages and the drop tests are described in [7], [8] and [9].

The sequence of a typical slap down impact is shown in *Fig.11* at the example ANF-18. The package was dropped from a height of 9 m in a 15° declined position. In *Fig.12* we see the corresponding and in principle for the most slap down impacts typical accelerometer signals of the package first end and slap down end. The according velocity-time curves, obtained by integration are shown in *Fig.13*. The container end which hits first the target was decelerated during a few milliseconds from the initial velocity 13 m/s to zero and remains in contact with the target, while the opposite end accelerates from initially 13 m/s to 21 m/s in a time period of 10 milliseconds. After 60 ms at time $t$ = 70 ms the casks opposite end hits with nearly 24 m/s in a slap down impact the target.

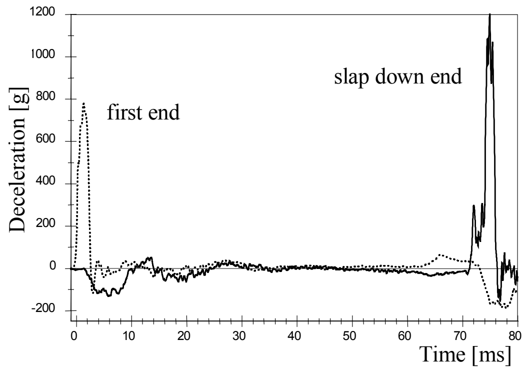


*Fig.12.* ANF-18. Deceleration signals.

The drop tests showed that sliding between the end of the cask hitting first does not occur during the impact (see also [10]). The impacts are rough. If the impact were frictionless (smooth impact) the first end would slip out under the falling cask and the cask would rotate about its center of mass (see *Fig.2*). Therefore the analytical results for the smooth impact have more a theoretical value. However the equations for the rough impact with $0 < k < 1$ are a suitable tool to describe in a good estimation the kinematic of the package in a real drop test situation.

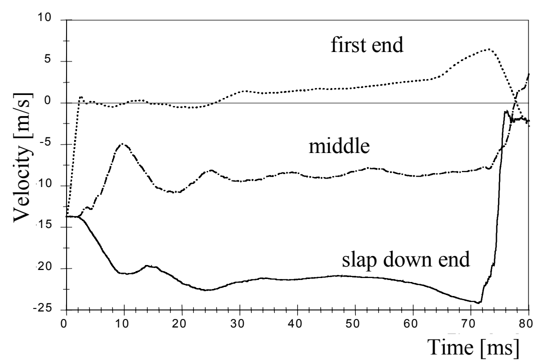


*Fig.13.* ANF-18. Velocity time curve.

The slap-down velocities of various packages taken from deceleration measurements in 9 m and 15° degree declined drop tests are compared with the analytical results in *Table 3*.


*Fig.11.* ANF-18. 9 m and 15° declined drop test.
a) Free Fall
b) First Impact
c) Slap-Down-Impact

| Cask | | | Experimental Results | | Analytical Results |
|---|---|---|---|---|---|
| Name | Geometry | Mass $m$ | slap-down velocity | qualitative specification of first impact | slap-down velocity |
| ESBB | $l$ = 4538 mm; Ø 150 mm | 315 kg | ≈ 25 m/s | rebound; $k > 0$ | rough, $k$ = 1: 25 m/s |
| ANF-10 | $l$ = 4725 mm; 667 mm x 565mm | 1429 kg | ≈ 23 m/s | rebound; $k > 0$ | rough, $k$ = 1: 25 m/s |
| ANF-18 | $l$ = 5512 mm; 960 mm x 792 mm | 4466 kg | ≈ 21-24 m/s | rebound; $k > 0$ | rough, $k$ = 1: 25 m/s |
| CASTOR VHLW | $l$ = 4486 mm; Ø 1156 mm | 20950 kg | ≈ 20 m/s | $k \to 0$ | rough, $k$ = 0: 20 m/s |

*Table 3.* Cask drop from a height of 9 m. Impact angel 15 degree. Comparison between experimental and analytical results.

The first impact caused a clearly rebound of the first three packagings so that for the comparison $k$ is set to 1 in the analytic calculation. For the CASTOR VHLW cask with its impact limiter $k$ is set to 0. The theoretical and measured slap down velocities are close together.

## SUMMARY

This paper describes a methodical way to find critical drop angles or better a range of drop angles for oblique drops of a packaging used for the transport of radioactive materials. In a first step the packaging is idealised as a rigid body which can have four different borderline cases of impact contact conditions (ideal elastic or ideal plastic impact, with or without friction between container and target during primary impact). This analytical model has the benefit that parameter studies can be done easily, i.e. by changing the degree of plasticity of the collision using the coefficient of restitution $k$ . A knowledge about the size of the contact force or the impact time is not necessary. Secondly, it is important to know the total amount of kinetic energy remained in the packaging shortly before the second impact happens. Both information, the range of useful drop angles and the remaining kinetic energy for the second impact, are important for a well-founded choice of a test drop angle or for doing a large-scaled three-dimensional numerical analysis of the structure loading in case of a slap down event.

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