# COMPUTER MODELLING FOR RISK ASSESSMENT OF TRANSPORTATION USING METHODS OF FUZZY SET THEORY

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### SUMMARY

Computer software for risk assessment of transportation of important freight has been developed. It incorporates models of transport accidents, including terrorist attacks. These models use, among the others, input data of cartographic character. Geographical information system technology and electronic maps of an area are involved as an instrument for handling this kind of data. Fuzzy set theory methods as well as standard methods of probability theory have been used for quantitative risk assessment. Fuzzy algebraic operations and their computer realisation are discussed. One preliminary example of risk assessment is described.

### INTRODUCTION

Computer modelling for risk assessment of emergency situations and terrorist attacks on railway transportation of important freight may be useful for choosing several possible routes or for allocating resources to safeguard valuable documents or materials, moved by convoy, against an overt terrorist attack. It is also useful for ranking the sensitive areas at a site according to their survivability of a given hypothesised terrorist attempt and to compare various defense strategies.

## DESCRIPTION OF COMPUTER MODELLING AND RISK ASSESSMENT

The probability of accidents can be estimated by using models of transport accidents and terrorist actions. In any case it is necessary both to develop models and to establish the rules to combine the data for different kind of accidents. It is not a simple problem and we will discuss it elsewhere. In the first step the following situations are taken into consideration as possible accidents in railway transportation in our system: 1) fall from a bridge, 2) fall from a switch, 3) fall from rails on railway with a double track, 4) fall from rails on railway with a single track.

The main problem is a quantitative risk assessment. First, it is important to understand the nature of information which we use in data processing. There are many types of uncertainty involved in the estimation of events such as transport accidents and (or) terrorist actions. The conventional approach of uncertainty treating is to apply statistical methods of estimation, which are, in turn, based upon the concept of probability.

Even in cases where the source of uncertainty is of a nonstatistical nature, formal application of statistical methods of analysis is often made to deal quantitatively with uncertainty, tacitly accepting the premise that uncertainty - whatever its nature - can be equated with randomness. Most of the work of risk analysis or risk assessment has been done using such methods. But nothing would be more misleading than to use the term "probability" or "probabilistic" when the term is not valid, since a number of nonprobabilistic uncertainties would be involved in some type of estimation. To avoid confusion, it would be better to restrict the use of probability in a scientific sense to repetitive events only, and to use the term "possibility" when we wish to speak of our expectation of nonrepetitive events, as suggested in fuzzy set theory (Nishiwaki et al. 1984).

Sometimes, an extremely low frequency accident is called a "low probability - high consequence" accident. However, if the frequency is extremely low, the event may not be considered repetitive and the concept of probability may not be applicable in the mathematical sense. There must be a minimum probability below which the mathematical theory of probability is not applicable. The consideration of such events with an extremely small probability but very large consequence poses special technique of risk assessment too. Fuzzy set theory permits us to treat this problem. Zadeh (Zadeh 1965) established fuzzy set theory to model fuzzy information. Instead of a characteristic function with binary meaning 0 or 1 which expresses nonfuzzy set A, he suggested the "membership function"  $\mu_A(x)$  to describe the degree of belongingness of the element x to a fuzzy set A. Thus, fuzzy value x is fully characterised by its membership function  $\mu_A(x)$ . The fuzzy analysis involves extended algebraic operations i.e., operations of fuzzy numbers according to Zadeh's extension principle (Zadeh 1973).

Let  $A_1, A_2, ..., A_N$  be the fuzzy sets defined on the universes  $X_1, X_2, ..., X_N$ , respectively. Let f be a function which maps  $X_1 \times X_2 \times ... \times X_N$  to Y. According to this principle, the fuzzy image B of  $A_1, A_2, ..., A_N$  through f has the membership function

$$\mu_{\mathbf{B}}(\mathbf{y}) = \bigvee_{\substack{\mathbf{x}_{1} \circ \mathbf{X}_{1}, \dots, \, \mathbf{x}_{N} \circ \mathbf{X}_{N} \\ \mathbf{y} = f(\mathbf{x}_{1}, \dots, \, \mathbf{x}_{N})}} \{ \wedge [\mu_{A^{1}}(\mathbf{x}_{1}), \dots, \mu_{A^{N}}(\mathbf{x}_{N})] \}, \quad (1)$$

where  $\lor$  and  $\land$  stand for max and min, respectively.

With the help of equation (1), any mathematical relations between non-fuzzy elements, including algebraic operations, can be extended to fuzzy sets defined on these elements. Following this principle the membership function  $\mu_3(z)$  of z is obtained from the membership

functions  $\mu_1(x)$  and  $\mu_2(y)$  of x and y according to

$$\mu_{3}(z) = \max \{\min [\mu_{1}(x), \mu_{2}(y)]\}, \qquad (2)$$

$$x * y = z$$

where the symbol \* denotes any of the algebraic operations  $+, -, \times$  and +. The extension principle provides a general solution to the processing of fuzzy algebra. The expression as given in equation (2) is deceptively simple. In practice, there is an infinitely large number of combinations of x's which give the same value y, and it is possible to conduct an exhaustive search of all the combinations to find the maximum membership.

Hence, implementation of the solution according to equation (2) is difficult for realistic problems in engineering even on a computer. There is not enough space to describe in details methods proposed to solve this equation. The most suitable method is the Vertex method proposed by Dong and Wong (Dong and Wong 1985). This method is based on the key idea of the non-linear programming approach. The key step is a statement that for a binary algebraic operation on two fuzzy numbers  $\mathbf{K} = \mathbf{I}^* \mathbf{J}$ , the optimum  $(\mathbf{x}, \mathbf{y})$  pair is selected by setting  $\mu_{\mathbf{I}}(\mathbf{x})=\mu_{\mathbf{J}}(\mathbf{y})$ . This condition, together with the algebraic relation  $\mathbf{x}^*\mathbf{y}=\mathbf{z}$ , determine the membership of the solution. Instead of performing these computations analytically to find values of  $\mathbf{z}$ , the idea can be made to work for values of  $\mu_{\mathbf{k}}(\mathbf{z})$ , where  $\mathbf{K}$  is the result desired. It's possible to work with  $\alpha$ -cut intervals of  $\mathbf{K}$  and relate them to the corresponding optimum  $\alpha$ -cut intervals of  $\mathbf{I}$  and  $\mathbf{J}$ . An algorithm and program were made to realise this method. An algorithm consists of the following steps.

1. Select an  $\alpha$  value where  $0 \le \alpha \le 1$ .

2. Find the intervals in X and Y which correspond to this  $\alpha$ . These are the  $\alpha$ -cuts of I and J, respectively.

3. The interval in K which corresponds to those of X and Y is computed. This interval is the  $\alpha$ -cut of K.

4. Repeat for different values of  $\alpha$  to fill in an  $\alpha$ -cut representation of the solution K. We worked out computer code for processing of fuzzy information on the basis of this algorithm.

The model of transport accidents uses, among others, the input data of cartographic character. The probabilities of transport accidents and their outcomes depend on the proximity of settlements, presence of bridges, tunnels and other features of transportation routes. Positions of all these objects with respect to each other and to a transportation route are very important for evaluation the probability of an accident. For handling this kind data base we use geographical information system technology and electronic maps of an area.

At the minimum configuration this is a one-layer map containing information on roads, railways and their characteristics. This map includes 3 classes of topographic objects:

1. A junction node: point of road intersections or forks, or any other point which has any definite logical significance (point of loading, unloading, transfer etc.).

2. A segment: part of a road between two border junction nodes (beginning and end of a segment).

3. A route: a sequence of segments when any two neighbouring segments have one common border node and each n-th segment is contiguous only with (n+1)-th and (n-1)-th segments.

On every route a set of distributed parameters is defined. In this work we used 10 parameters: permissible velocity, number of tracks, density of switches (number of switches per unit route length), density of bridges, density of tunnels, proximity of settlements, proximity of forests, proximity of dangerous objects (storage of explosive, fire-dangerous or poisonous substances), complexity of area relief, response guard force accessibility.

Preparation of calculation input data consists of several actions:

1. Preparation of an electronic road map. A paper map of an area with roads is converted to digital form with the help of a digitizer. Another way is to transform it to a raster image and digitize it by means of special software.

2. Constructing a route set for freight transportation from starting-point to destination. It is done on a vector electronic map of roads with the help of special software.

3. Definition of values of all parameters for each route. For this operation a route is considered as a set of nonoverlaping intervals. The value of a parameter is defined to be constant on an interval. A user can define this value by hand or, if the value is presented as a function of the route point, it is calculated automatically. Automatic calculation can be done easily when the parameter depends only on the proximity of any objects (forest, dangerous object, guard forces base, etc.), but this requires multilayer electronic maps.

The results of calculations are graphically represented as a function of route point on the map or as a special diagram. Such an approach allows us to choose several possible routes of transportation, compare them on the basis of different criteria and to draw the model results on the map of the area.

Using this computer system we calculated the risk of an emergency situation in transportation of nuclear power space installations on railways from Moscow to Turatam station in Kazakhstan. In the first stage we took into account only five potential sources of an accident. We used estimated data of probabilities  $p_i$  of these dangers and their maximum  $p_i^{\text{max}}$  values obtained by processing data for trouble-free transportation of nuclear power space installations on railways (Volkov U.V., et.al), assuming that each number from these data is a fuzzy number which fits a Simpson distribution with mean value  $\overline{p}_i = p_i$ , maximum value  $p_i^{\text{max}}$  and minimum value  $p_i^{\text{min}}$ . The value  $p_i^{\text{min}}$  was taken from the equation  $\overline{p}_i / p_i^{\text{max}} - p_i^{\text{min}} / \overline{p}_i$ . This assumption about the fuzziness of  $p_i$  is in good agreement with the nature of information on the dangers of transportation because the conclusion about possible transport accidents is based on trouble-free experience in transportation of nuclear power space installations on railways. We propose the following equation as an inference rule to relate the total risk of a transport accident with its components

$$\mu_{\rm R} = \sum_{i} \mu_{i} - \sum_{i,j} \mu_{i}\mu_{j} + \sum_{i,j,k} \mu_{i}\mu_{j}\mu_{k} + \dots + (-1)^{n-1} \mu_{1}\mu_{2} \dots \mu_{n}, \qquad (3)$$

where  $\mu_R$  is the membership function of the total risk of transport accidents,  $\mu_i$ ,  $\mu_j$ ,  $\mu_k$  are the membership functions of the components; i, j, k = 1, 2, ..., n; in our case n=5.

In the calculations:  $p = 0.43 \times 10^{-2}$ ,  $p_1^{max} = 1.45 \times 10^{-2}$ ,  $p_1^{min} = 0.12 \times 10^{-2}$ ;  $p_2 = 0.43 \times 10^{-2}$ ,  $p_2^{max} = 1.45 \times 10^{-2}$ ,  $p_2^{min} = 0.12 \times 10^{-2}$ ;  $p_3 = 0.43 \times 10^{-2}$ ,  $p_3^{max} = 1.45 \times 10^{-2}$ ,  $p_3^{min} = 0.12 \times 10^{-2}$ ;  $p_4 = 0.42 \times 10^{-2}$ ;  $p_4^{max} = 1.94 \times 10^{-2}$ ,  $p_4^{min} = 0.09 \times 10^{-2}$ ;  $p_5 = 0.42 \times 10^{-2}$ ,  $p_5^{max} = 1.44 \times 10^{-2}$ ,  $p_5^{min} = 0.12 \times 10^{-2}$ ,  $p_5^{max} = 0.12 \times 10^{-2}$ . Operations on fuzzy numbers were performed using equation (2). It is necessary to note that we did not make any assumption about  $p_2$ . Details of the computations will be omitted, but the result - membership function of risk of transport accidents is given in fig. 1.

Maximum value of the membership function,  $\mu_{R}$ , corresponds to the possible value of risk of transportation,  $2.14 \times 10^{-2}$ . Minimum value of risk is  $2.14 \times 10^{-3}$  and maximum value of risk is  $7.73 \times 10^{-2}$ . The width of the curve describes the possible interval of values of risk. It is due to fuzziness of the initial information.



Risk assessment of terrorist attacks was simulated by the analysis of threat level during transportation. So, we envisage a target whose vulnerability changes with time. We defined success by the terrorists as hands-on unauthorised access to the freight. Our model assumed that the guard force accompanies and protects the target and that there exists a response force that will respond upon notification of the attack (Martz and Johnson 1987). It is not always true that the route with the greater vulnerability will also have the smaller survivability. Thus it is possible to manage target vulnerability by allocating safeguard resources.

Using our computer system we estimated vulnerability of the route from Moscow to Turatam. We considered the probability that an attack occurs during the i-th interval. From the Poisson process assumption, the probability was approximately given by (Martz and Johnson 1987)

$$P(A) = 1 - \exp(-\sum_{i} \lambda_{i} t_{i}) = 1 - \exp(-\lambda \sum_{i} Z_{i} t_{i}), \quad (4)$$

where A indicates a terrorist attack sometime during the entire operation;  $A_i$  indicates a terrorist attack during the i-th interval, i=1,2,...,n; t<sub>i</sub> is the average time to complete the i-th activity;  $Z_i$  is the subjective threat level during the i-th activity ( $Z_i = 1, 2, ..., 10$ );  $\lambda_i$  is the threat intensity associated with the i-th activity (threats/unit time);  $\lambda$  is the nominal threat intensity (threats/unit time). In a suggestion that  $\lambda = 0.84$  incidents per month (Martz and Johnson 1987), we made an assumption that  $Z_i = 2$  for railway transportation on a railway with a double track,  $Z_i = 2.5$  for railway transportation on a railway with a single track and  $Z_i = 9$  for

a bridge. The estimated probability of the terrorist attack was  $P(A) = 1.08 \times 10^{-2}$ . This high level of probability of terrorist attack can be explained by a rather high average intensity of incidents of terrorism. Unfortunately we do not know what part of this activity is engaged in railway terrorism. The estimation of survivability on the route can be made by using information about the safeguards, convoy guard force characteristics, response force characteristics, terrorist characteristics and engagement characteristics. The probability of a successful terrorist attack is quite sensitive to the number of guards. For example, for the anticipated small-sized terrorist groups 5 guards yield significantly greater survivability of attack than 2.

### CONCLUSION

The computer software will be further developed to achieve optimum strategy for transportation of different important freight. First of all, further study and development of a model of transport accidents will be done. Further investigation and development of mathematical methods of computations for more accurate quantitative assessment of different kinds of risk should be done. We believe that, following this method, an intelligent system for transportation can be worked out.

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