Comparison of Elastic and Inelastic Analyses*

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INTRODUCTION

The use of inelastic analysis methods instead of the traditional elastic analysis methods in the design of radioactive material (RAM) transport packagings leads to a better understanding of the response of the package to mechanical loadings. Thus, better assessment of the containment, thermal protection, and shielding integrity of the package after a structural accident event can be made. A more accurate prediction of the package response can lead to enhanced safety and also allow for a more efficient use of materials, possibly leading to a package with higher capacity or lower weight. This paper will discuss the advantages and disadvantages of using inelastic analysis in the design of RAM shipping packages.

The use of inelastic analysis presents several problems to the package designer. When using inelastic analysis the entire nonlinear response of the material must be known, including the effects of temperature changes and strain rate. Another problem is that there currently is not an acceptance criteria for this type of analysis that is approved by regulatory agencies. Inelastic analysis acceptance criteria based on failure stress, failure strain, or plastic energy density could be developed. For both elastic and inelastic analyses it is also important to include other sources of stress in the analyses, such as fabrication stresses, thermal stresses, stresses from bolt preloading, and contact stresses at material interfaces.

Offsetting these added difficulties is the improved knowledge of the package behavior. This allows for incorporation of a more uniform margin of safety, which can result in weight savings and a higher level of confidence in the post-accident configuration of the package. In this paper, comparisons between elastic and inelastic analyses are made for a simple ring structure and for a package to transport a large quantity of RAM by rail (rail cask) with lead gamma shielding to illustrate the differences in the two analysis techniques.

ANALYSIS OF A SIMPLE RING STRUCTURE

A very simple structure (a ring impacting a block of foam) was chosen to illustrate the differences between elastic and inelastic analyses and between equivalent static and dynamic analyses. This simple ring structure is shown in Figure 1. Material properties consistent with actual tensile test results of an A516-Gr60 pressure vessel steel were chosen for the ring. This material has a clearly defined yield plateau with significant strain hardening. Because the purpose of this study was to determine the differences between elastic and inelastic analysis methods, the actual yield (268 MPa) and ultimate stress (465 Mpa) values from the tensile test were used rather than the tabulated minimum

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values that would normally be used in design. The allowable stress using the elastic criterion of Regulatory Guide 7.6 (U.S. NRC 1978) is 419 MPa. Similarly, the allowable stress using the inelastic criterion from the ASME Boiler and Pressure Vessel Code (ASME 1983) is 326 MPa.

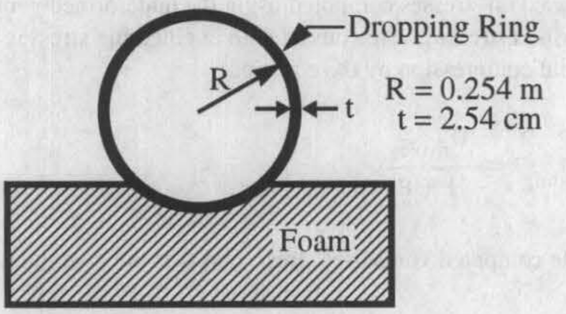


Figure 1. Simple Ring Structure

Five different analyses were performed on this structure. First, an equivalent static handbook (Roark and Young 1975) analysis was performed. In this analysis, an energy balance between the potential energy of the ring before it was dropped from a 9 m height and the strain energy of the foam was used to determine the maximum foam crush. To calculate the stress in the ring, two primary assumptions were made: (1) the foam provides a uniform pressure equal to its crush strength over the area of contact with the ring and (2) the force in the foam is in equilibrium with the inertia of the ring. Multiplying the ring's maximum footprint in the foam (easily computed from the depth of crush above) by the crush strength of the foam gives a maximum applied force of 29.2 kN. This maximum force generates a peak deceleration of 3827 m/s^2 or 390 g's. The crush depth, the footprint in the foam, and the maximum stress in the ring are all shown in Table 1.

The next two analyses were performed with the finite element program SANTOS (Stone 1992), which computes the nonlinear quasistatic response of solids by the dynamic relaxation method. The same problem as described above for the handbook solution was solved using SANTOS. The foam was not modelled, but was replaced by a pressure over the same area as for the handbook solution above. One analysis utilized a linear elastic material response for the ring and the other an inelastic response, where the strain hardening was characterized by the plastic strain raised to a fractional power (Stone et al. 1990). The maximum stresses computed are again shown in Table 1.

The final two analyses employed a nonlinear transient-dynamic finite element program, PRONTO2D (Taylor and Flanagan 1987). As above, the ring was modelled as an elastic material in one analysis and as an inelastic material in the other. The foam was modelled using a recently developed phenomenological plasticity theory (Neilsen et al. 1986). The analyses commenced with the ring just in contact with the foam block. The ring was given an initial velocity consistent with a 9 m drop (13.4 m/s). The analysis was carried out past minimum kinetic energy; the ring had started to rebound from the foam. The maximum depth of foam crush, the ring footprint in the foam, the maximum stress in the ring, and the maximum net force during the impact are shown in Table 1.

Table 1. Results from the Analysis of a Ring Dropping onto Foam

	Depth of Crush	Footprint in Foam	Max Stress in Ring	Maximum Force
Closed Form	4.83 cm	21.06 cm	419 MPa	29.2 kN
Static Elastic	n. a.	n. a.	423 MPa	29.2 kN
Static Inelastic	n. a.	n. a.	310 MPa	29.2 kN
Dynamic Elastic	3.73 cm	28.96 cm	427 MPa	21.8 kN
Dynamic Inelastic	3.68 cm	28.96 cm	309 MPa	21.8 kN

The stresses computed for the elastic analyses were very similar to each other, as were the stresses for the inelastic analyses. The elastically computed stresses were within two percent of the elastic stress criterion of Regulatory Guide 7.6, as expected. The stresses computed using inelastic material response were approximately 5 percent below

The finite element model for the rail RAM transportation package 9 m corner drop scenarios consisted of a total of 31,960 elements with two elements through the thickness of the inner shell and two elements through the thickness of the outer shell. All analyses for the corner drop impact scenario were performed with a transient dynamic analysis code PRONTO3D (Taylor and Flanagan 1989). This code calculates stresses/strains based on the deformed geometry. The criterion of NRC Regulatory Guide 7.6 (U.S. NRC 1978) and ASME Boiler and Pressure Vessel Code, Section III, Appendix F (ASME 1983) are based on stresses computed using the undeformed geometry (engineering stress). Therefore the computed von Mises stresses were converted to engineering stresses by conservatively assuming that all strains were uniaxial compression by the equation:

$$\sigma_{\mathrm{eng}} = \frac{\sigma_{\mathrm{mises}}}{1-\varepsilon}$$

where σ_{eng} is the engineering stress, σ_{mises} is the computed von Mises stress, and ε is the computed strain.

The 9 m center-of-gravity-over-corner drop impact was modelled as a dynamic event with initial velocity of 13.4 m/s. Figure 3 shows the deformed shape of the rail cask for the inelastic analysis.

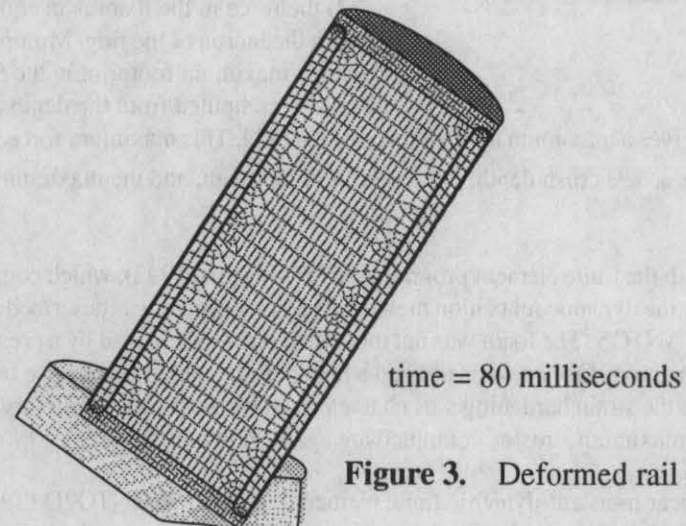


Figure 3. Deformed rail cask after 9 m corner drop

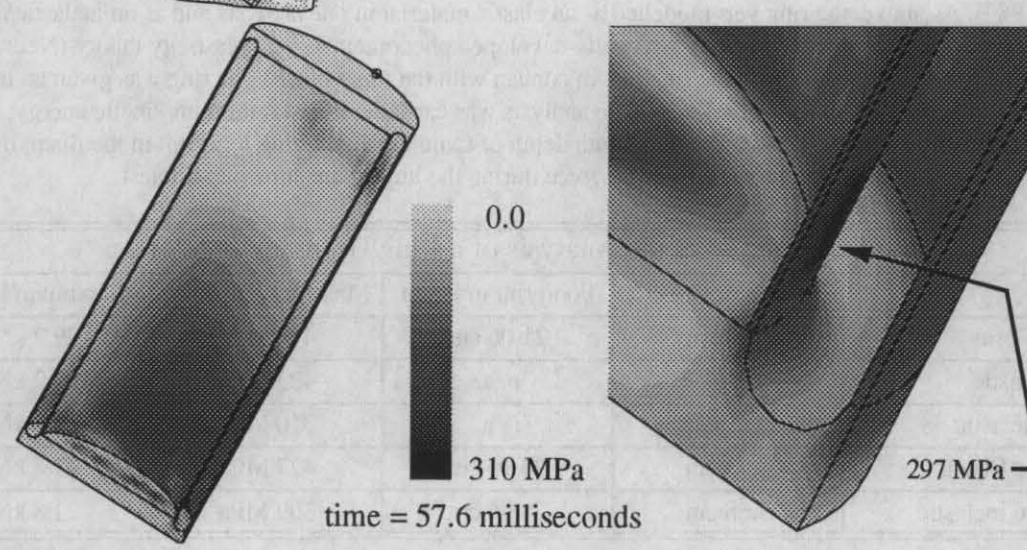


Figure 4. Maximum von Mises stress during the 9 m corner drop of inelastic rail cask

the ASME Boiler and Pressure Vessel criteria. The major difference in stress values between the static and the dynamic analyses was that the maximum stress occurred at the maximum foam crush for the static analyses while the maximum stress occurred at about half the time to minimum kinetic energy or half the time to maximum foam crush for the dynamic analyses. For the dynamic analyses the maximum stress in the ring occurred at an earlier time than the maximum load. The dynamic analyses developed a larger footprint with a corresponding lesser depth of crush for approximately the same energy absorbed in the foam. Much of this difference is due to the dynamic analysis taking into account the deformation of the ring, while the static analyses assumed the foam loaded an undeformed ring.

ANALYSIS OF A RAIL CASK WITH LEAD GAMMA SHIELDING

In this section the problems and benefits of using elastic and inelastic analysis in the design of RAM transportation packages are explored via a design for shipping a bulk quantity high level RAM waste. The waste is assumed to have very little strength but high volumetric stiffness and a specific weight of 1.7. It is assumed that the shielding requirements for the package are similar to those for spent fuel. The package is a rail cask that utilizes lead for its gamma shielding, 304 stainless steel shells on the inside and outside of the gamma shielding, and solid stainless steel ends as shown in Figure 2. In addition, the package is encased in neutron shielding, a 304 stainless steel neutron shielding shell and 0.32 g/cm^3 polyurethane foam impact limiters. The dimensions and material properties for the rail cask can be found in prior work by the authors (Heinstein and Ammerman 1992). This reference has detailed analyses for the rail cask as well as a smaller package for transporting RAM by truck (truck cask).

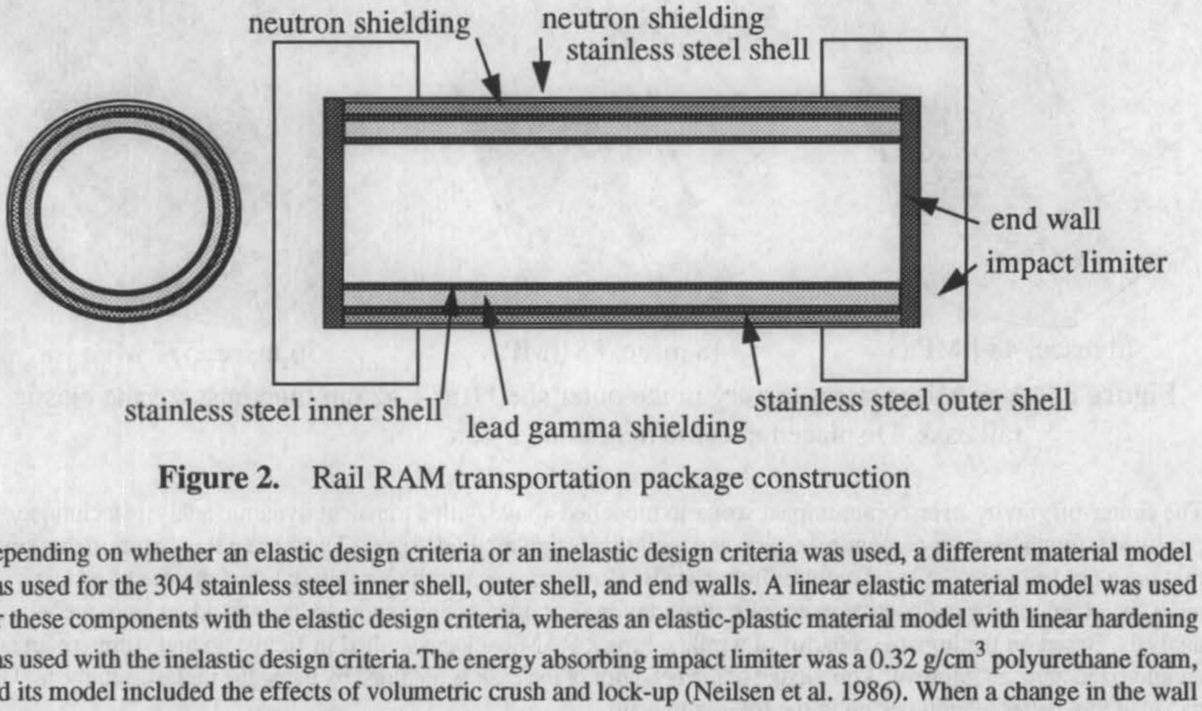


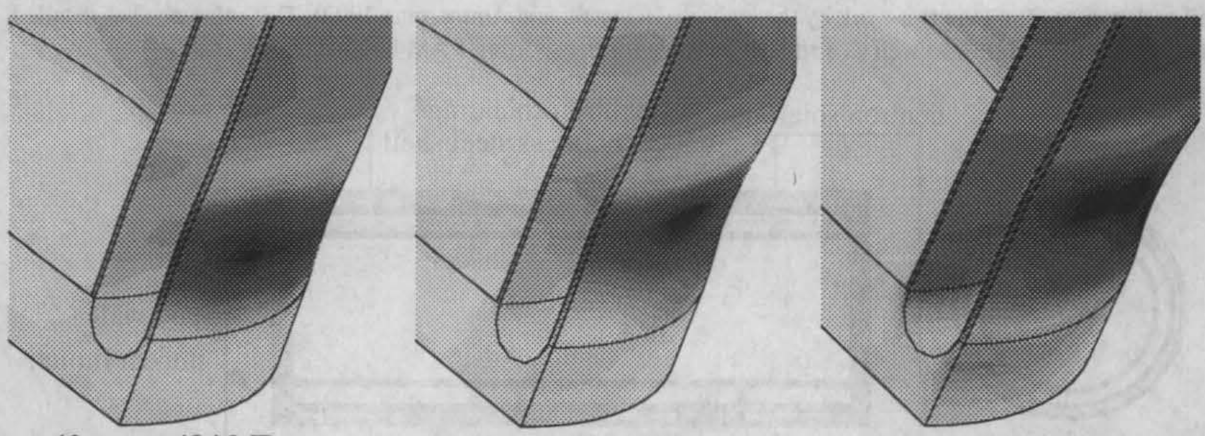
Figure 2. Rail RAM transportation package construction

Depending on whether an elastic design criteria or an inelastic design criteria was used, a different material model was used for the 304 stainless steel inner shell, outer shell, and end walls. A linear elastic material model was used for these components with the elastic design criteria, whereas an elastic-plastic material model with linear hardening was used with the inelastic design criteria.The energy absorbing impact limiter was a 0.32 g/cm^3 polyurethane foam, and its model included the effects of volumetric crush and lock-up (Neilsen et al. 1986). When a change in the wall thickness was required, a replacement ratio of 1 part lead to 1.75 parts stainless steel was used such that the shielding effectiveness was unchanged.

The maximum allowable stresses are computed by the formulas specified in the NRC Regulatory Guide 7.6 (U.S. NRC 1978) for the elastic analysis, and in the ASME Boiler and Pressure Vessel Code, Section III, Appendix F (ASME 1983) for the inelastic analysis. For the stainless steel material, the maximum allowable membrane plus bending stress was 482 MPa for the elastic analysis, and 465 MPa for the inelastic analysis. No design changes were made in the elastic analyses based on buckling according to the ASME Boiler and Pressure Vessel Code, Case N-284 (ASME 1980).

In the inelastic analysis, the von Mises stress increases in the outer shell as the cask is loaded to the maximum g-load. Because the stainless steel is allowed to yield, part of the load is transferred to the shielding and inner wall. The maximum von Mises stress during the corner drop event, was 297 MPa (engineering stress of 308 MPa) at 57.6 milliseconds. The location of this maximum stress, as shown in Figure 4, was in the inner shell. For the inelastic analysis, a plastic strain of 0.063 for the 304 stainless steel was observed in the inner shell of the cask.

Figure 5 shows a series of deformed shapes (with displacements magnified by 5x) of the outer shell (for a cask design with outer shell thickness 1.52 cm) at 40 msec, 48 msec, and 56 msec for an elastic analysis. The high stresses are due to a combination of the endwall bending the shell and the impact limiter pushing inward on the outer shell. Note that the outer shell thickness of 1.52 cm is the same used in the inelastic analysis. The outer shell thickness was significantly increased in the redesigns (to 8.89 cm), yet the maximum stress still exceeded the allowable stress. With the outer shell thickness of 8.89 cm the maximum von Mises stress was 598 MPa (engineering stress of 599 MPa) at 59.2 milliseconds which corresponds to the maximum g-loading on the cask. The location of this maximum stress was in the outer shell as shown in Figure 6. Because of the relatively small stiffness of the lead shielding, practically none of the load on the outer shell is transferred to the inner shell. The maximum von Mises stress of 598 MPa *exceeds* the maximum allowable membrane plus bending stress of 482 MPa specified by the NRC Regulatory Guide 7.6. The outer shell wall thickness was increased from an initial thickness of 1.52 cm to a point where it was felt that the design was no longer realistic and, therefore, no further redesign was attempted.

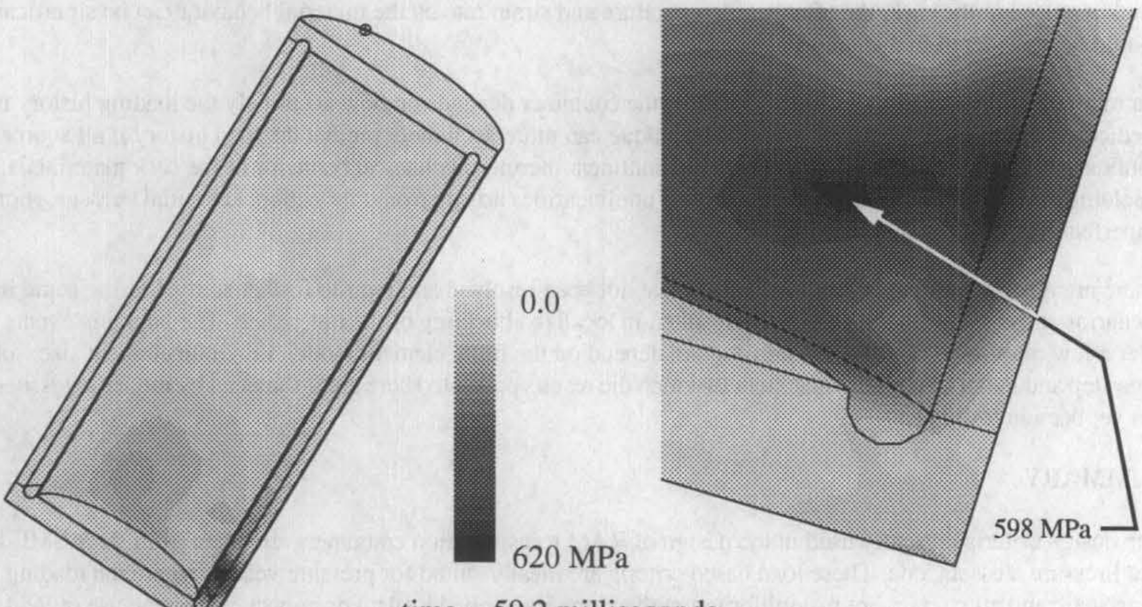


40 msec, 484 MPa 48 msec, 851 MPa 56 msec, 577 MPa

Figure 5. Von Mises stress history in the outer shell (for 1.52 cm thickness) of the elastic rail cask. Displacements are magnified by 5x

The center-of-gravity-over-corner impact scenario modelled above with a transient dynamic analysis technique provided a foundation for comparing elastic and inelastic design methodologies. There are a few issues in this study that have not been resolved and require further study. However, even with these limitations, the use of inelastic analysis technique for radioactive material transportation container design seems to have an advantage over elastic analysis. Based on the impact scenarios of a rail and truck RAM package studied in Heinstein and Ammerman and summarized here, an improved knowledge of the behavior of the cask is obtained by using the inelastic analysis. This can lead to a better overall design in the following ways.

First, elastic analysis may underpredict maximum stress at a particular location, resulting in inappropriately sized wall sections. Elastic analysis does not properly account for the decrease in stiffness resulting from yielding in part of the structure and does not show the redistribution of load caused by this yielding. This was found to be the case in the 9 m end drop impact of the rail cask. The maximum stress predicted in the elastic analysis was 276 MPa whereas the maximum stress in the inelastic analysis was 496 MPa. This was a result of the outer shell yielding and redistributing the load to the gamma shielding and inner shell. It was also observed in the inelastic analysis that significant plastic straining can occur through the thickness in several areas. This may indicate that the elastic analysis is neglecting significant physical features of the impact scenario.



time = 59.2 milliseconds

Figure 6. Maximum von Mises stress during the 9 m corner drop of elastic rail cask (for 8.89 cm outer shell thickness)

Second, elastic analysis may overpredict the maximum stress. The inelastic shells can yield and redistribute the loading to other less loaded parts of the structure, whereas the elastic shells cannot predict this behavior. This was shown in the 9 m center-of-gravity-over-corner drop of the rail cask. Based on the elastic analysis of the impact event, an outer shell thickness of over 8.89 cm would be required to meet the design criteria. With the same impact limiter, the inelastic analysis suggested that the loading on the outer shell causes it to yield and redistribute the load to the gamma shielding and inner shell requiring an outer shell thickness of only 1.52 cm. Furthermore, it was observed in the truck cask analyses that the amount of stress redistribution can be small and still influence the location, and time of occurrence of the maximum stress. Therefore, the inelastic analysis may also allow for a better distribution of structural material - which can lead to weight savings. The weight savings can increase the capacity of the package, thereby decreasing the number of shipments required to transport a given quantity of material, which increases the overall shipping program safety. The use of inelastic analysis may also decrease the overall cost of a transportation package, especially for designs where multiple packages will be constructed.

ISSUES INVOLVED IN CONDUCTING ACCURATE ANALYSES

The use of inelastic analysis for RAM transportation containers potentially has several advantages over the currently used elastic analysis. The most prominent of these is that the analysis method models the behavior of the package more closely which leads to a better understanding of the response of the container to the loads applied to it. The transient dynamic analysis technique utilized in this study provides improved knowledge of the structural integrity of the cask, but with additional cost. The computer cost for one center-of-gravity-over-corner impact scenario summarized here involved approximately 25 cpu-hours on a Cray YMP. This cost should be added to the time spent by an experienced user in constructing the finite element model. Such a model typically includes a variety of material models and nonlinear material behavior.

Some additional material properties required include strain rate and temperature dependent stress-strain curves. In the examples considered in (Heinstein and Ammerman 1992) and summarized here, the strain rates can typically range from $10^{-1}\ s^{-1}$ to $10^{3}\ s^{-1}$. The fact that the contents will have a temperature higher than the outside ambient means there will be a temperature gradient through the wall of the cask. For certain materials, especially the lead shielding used in the cask, the effects of temperature and strain rate on the material behavior can be significant and should be considered in the analysis.

An improved understanding of the response of the container depends on how accurately the loading history is predicted. The transient dynamic analysis technique can more accurately predict the load history if all sources of nonlinearity are considered. That includes the nonlinear thermo-mechanical behavior of the cask materials, i.e. shielding, contents, and impact limiters, and the nonlinearities arising from fabrication, i.e. initial stresses, geometric imperfections, and fastener details.

There are also several modelling issues that have not been resolved and require further study. During some impact scenarios stress waves in the shell walls resulted in localized buckling of the inner shell. The buckling events occur over a few microseconds and, to some degree, depend on the finite element model, i.e. finite element size, solution time step and material model. The extent to which the results presented here are influenced by modelling issues have not yet been investigated.

SUMMARY

The design criteria currently used in the design of RAM transportation containers are taken from the ASME Boiler and Pressure Vessel Code. These load based criteria are ideally suited for pressure vessels where the loading is quasistatic and all stresses are in equilibrium with externally applied loads. For impact events, the use of load based criteria is less supportable. Impact events tend to be energy controlled, and thus, energy based criteria would appear to be more appropriate. Determination of an ideal design criteria depends on what behavior is desired. If the intent is that there will be no yielding in the package, an elastic analysis with an allowable stress less than the yield point stress is sufficient. This type of acceptance criteria will lead designers to using materials with the highest possible yield stress, and perhaps a lower margin of safety against gross rupture. However, if the goal is to prevent release of radioactive material, some amount of inelastic deformation is acceptable. In this case, the acceptance criteria should limit through wall tearing and keep deformations to an acceptably small amount. An elastic analysis cannot predict the margin of safety against through wall tearing and the deformations associated with an impact event nearly as well as an inelastic analysis. For the simple ring structure studied here, there is only about a 5 percent difference between the use of linear-elastic criteria versus inelastic criteria. Even the introduction of dynamics does not appreciably affect the stresses in the ring. However, the deformations in the foam (impact limiter) are different between the quasistatic and the dynamic analyses. For more complicated structures, such as the rail cask, the use of an equivalent uniform acceleration over the structure is difficult to justify. More importantly, equivalent static analysis is incapable of resolving the magnitudes and distributions of the load transfer between the impact limiter and the structure, where both strength and inertia are important. The overwhelming advantage of nonlinear dynamic analysis techniques is a better understanding of the response of the structure to the imposed environment. A better understanding of package behavior during impact events should lead to a safer package.

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