Development of a Phenomenological Constitutive Model for Polyurethane Foams

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INTRODUCTION

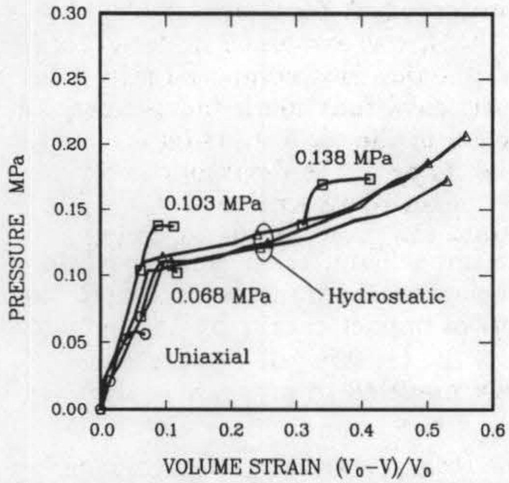
Rigid, closed-cell, polyurethane foam is used in impact limiters in nuclear waste transport containers. During a hypothetical nuclear waste transport accident, the foam is expected to absorb a significant amount of impact energy by undergoing large inelastic volume reductions. Consequently, the crushing of polyurethane foams must be well characterized and accurately modeled to properly analyze a transport container accident.

At the request of Sandia National Laboratories, a series of uniaxial, hydrostatic and triaxial compression tests on polyurethane foams were performed by the New Mexico Engineering Research Institute (NMERI). The combination of hydrostatic and triaxial tests was chosen to provide sufficient data to characterize both the volumetric and deviatoric behaviors of the foams and the coupling between the two responses. Typical results from the NMERI tests are included in this paper. A complete description of these tests can be found in Neilsen et al., 1987.

Constitutive models that have been used in the past to model foam did not capture some important foam behaviors observed in the NMERI tests. Therefore, a new constitutive model for rigid, closed-cell, polyurethane foams was developed and implemented in two finite element codes. Development of the new model is discussed in this paper. Also, results from analyses with the new model and other constitutive models are presented to demonstrate differences between the various models.

EXPERIMENTAL NMERI TESTS

Six different General Plastics foams varying in density from 0.032 gm/cm^3 (2 lb/ft^3) to 0.080 gm/cm^3 (5 lb/ft^3) were characterized in the NMERI tests. Uniaxial, hydrostatic and triaxial compression tests were performed. Results from a series of tests on 0.032 gm/cm^3 foam are presented in Figure 1, where both volumetric and axial stress-axial strain responses are shown. These results indicate that the mean stress at which volumetric yielding occurs is dependent on the deviatoric stress. For example, the mean stress at yield under hydrostatic loading is approximately 0.10 MPa, whereas the mean stress at yield is 0.06 MPa for uniaxial loading and 0.14 MPa for the triaxial test with a confining pressure of 0.103 MPa. In the triaxial test with a confining pressure of 0.138 MPa, the foam actually yields twice, once at 0.10 MPa during the hydrostatic phase of the test and then again at 0.17 MPa. For this triaxial test, the data indicate that when the additional axial loads are finally applied, the foam has higher resistance to the axial loads than to continued hydrostatic loading. This type of behavior is not commonly observed and is an indication of the unusual coupling between the volumetric and shear responses of the foam. The unloading is not shown, but is generally along a slope parallel to the initial loading curve with some additional unloading strain at very low stress levels.



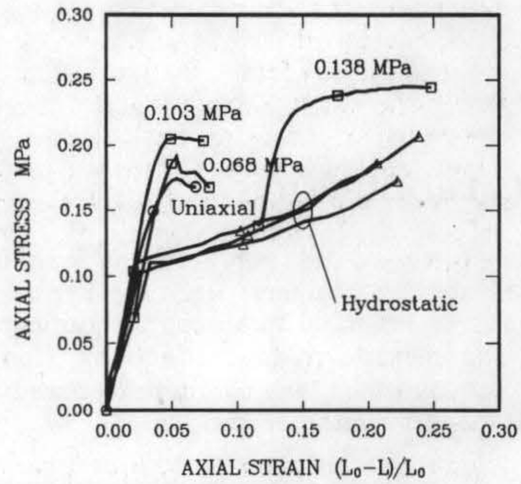


Figure 1: Results from the NMERI Tests on 0.032 gm/cm^3 Foam - Triaxial Specimens were Hydrostatically Loaded to the Labeled Pressure, then Axial Load Alone was Increased.

APPLICATION OF EXISTING CONSTITUTIVE MODELS TO POLYURETHANE FOAMS

The logical first step in the development of a constitutive model for polyurethane foams was to try to fit existing constitutive models to the test data. In this section, two multiaxial models which have been used in the past to model polyurethane foam behavior are evaluated with respect to the NMERI data. The two models considered in this section include: a conventional deviatoric plasticity model, and a cap model which combines volumetric plasticity with pressure dependent deviatoric plasticity.

Conventional Deviatoric Plasticity Model

Conventional deviatoric plasticity models were originally developed to model the response of metals but have been used to model polyurethane foam. The uniaxial yield strength of foam can be measured and extrapolated to multiaxial conditions using conventional deviatoric plasticity assumptions. One of the assumptions which must be evaluated, however, is that the model allows only elastic volume strains. The hydrostatic data in Figure 1 indicates that polyurethane foams undergo large plastic volume strains when subjected to loads of interest. Thus, conventional deviatoric plasticity models fail to capture the dominant energy dissipation mechanism of polyurethane foams, their plastic volumetric behavior. Another assumption made with conventional plasticity models is that the volumetric and deviatoric responses are not coupled. If a volumetric-deviatoric decomposition were valid, all of the volumetric responses in Figure 1 would be the same regardless of the load history. The curves in Figure 1 indicate that the volumetric response is clearly dependent on load history. Thus, conventional deviatoric plasticity models fail to capture two important features of polyurethane foam behavior: volumetric plasticity and volumetric-deviatoric coupling.

Cap Model

Cap models, multiaxial models which combine volumetric plasticity with pressure dependent deviatoric plasticity, were also investigated. A particular model of this type, developed for soil and concrete (Krieg, 1972), was examined in detail for its applicability to foam. In this model, the yield function is decomposed into deviatoric and volumetric parts. The volumetric yield function is independent of the deviatoric stresses, but the deviatoric portion of the yield function is dependent on the mean stress or pressure. The shape of the deviatoric yield surface is a paraboloid of revolution about the mean stress axis. The volumetric, Φ_v, and deviatoric, Φ_s, yield functions are given by the following equations

$$\Phi_v = p - f(\gamma) \tag{1}$$

$$\Phi_s = J_2 - (a_0 + a_1 p + a_2 p^2) \tag{2}$$

where p is the pressure or first invariant of the total stresses, γ is the engineering volume strain, f is a function defining the volumetric behavior, J_2 is the second invariant of the deviatoric stresses, and a_0, a_1 and a_2 are material constants. This model captures the volumetric plasticity of polyurethane foam. However, in this model, the volumetric response is independent of the deviatoric response. This assumption is not valid for the polyurethane foam data presented in Figure 1; otherwise, all the test data would coincide in the volumetric response plots. Neither conventional deviatoric plasticity models, nor cap models are appropriate for polyurethane foams. Therefore, a new constitutive model was developed.

MODEL DEVELOPMENT

The first step in the development of a new constitutive model for polyurethane foams was to examine the individual components of the foam structure. Each of the foams used in the NMERI tests was a closed cell foam with air inside the cells. Therefore, each foam consisted of two structural components: (1) the polymer structure or skeleton and (2) the air inside the foam. In applications where the air cannot escape from the skeleton during loading, the air can carry a substantial part of the load. In all of the NMERI tests except the uniaxial tests, the samples were jacketed and air could not escape. Thus, a model which considered the contribution of the air to the overall structural response of the foams was appropriate for the foam behavior exhibited in the NMERI tests.

Total foam response can be decomposed into the response of the skeleton and the response of the air in the following manner. Since the air cannot support shear stresses, the air contribution is completely volumetric. For convenience, the skeleton is assumed to occupy the same space as the foam. This implies that the skeleton and foam strain components are equal. Also, the foam stress components, σ_{ij}, are given by the following equation

$$\sigma_{ij} = \sigma_{ij}^{sk} + \sigma^{air}\delta_{ij} \tag{3}$$

where σ_{ij}^{sk} are the skeleton stress components and $\sigma^{air}\ \delta_{ij}$ represents the air contribution to the normal stress components. To better understand this equation, consider a hydrostatic compression test in which the foam sample is jacketed and the air is not allowed to escape. If the skeleton was structured such that it could not carry any load then the external pressure applied to the foam would equal the internal air pressure. In other words, the foam stress components would equal the air contribution. This foam would not be able to resist any deviatoric loading. In most foams, however, the skeleton is structured such that it can carry load and the contribution of the skeleton must be added to the air contribution to determine how much load the foam can carry.

The ideal gas law was used to derive the following expression for the air contribution,

$$\sigma^{air} = \frac{p_0[\gamma + (1-\phi)(1 - T_1/T_0)]}{(\gamma + 1 - \phi)} \tag{4}$$

where p_0 is the absolute internal air pressure when no load is applied to the foam, ϕ is the volume fraction of solid material, γ is the engineering volume strain, T_0 is the initial absolute temperature, and T_1 is the current absolute temperature. For isothermal conditions, the air contribution is equal to zero when the foam volume strain is equal to zero. For applications in which the air can escape from the foam, the load carried by the air can be neglected by setting p_0 equal to zero.

The skeleton response can now be determined from the NMERI tests. Since the foam and the skeleton occupy the same volume, the foam and skeleton strains are the same. Also, the skeleton stress components are determined by subtracting the expression given by Equation 4 for the stress carried by the air from the foam normal stress components given in Figure 1. The skeleton responses found in this way for 0.032 gm/cm^3 foam are shown in Figure 2.

The skeleton responses shown in Figure 2 indicate that for hydrostatic loading the yield stress can be expressed as a function of the volume strain, γ. If the loading is deviatoric, the axial yield stress appears to be equal to the axial yield stress for hydrostatic loading plus a constant. The NMERI test results also indicate that the skeleton response in a principal stress direction is not affected by the other principal skeleton stresses. Thus, the yield stress in each principal stress direction can be expressed as

$$g = A|II'| + B + C\gamma \tag{5}$$

where II' is the second invariant of the deviatoric strains, $|*|$ is the Heaviside

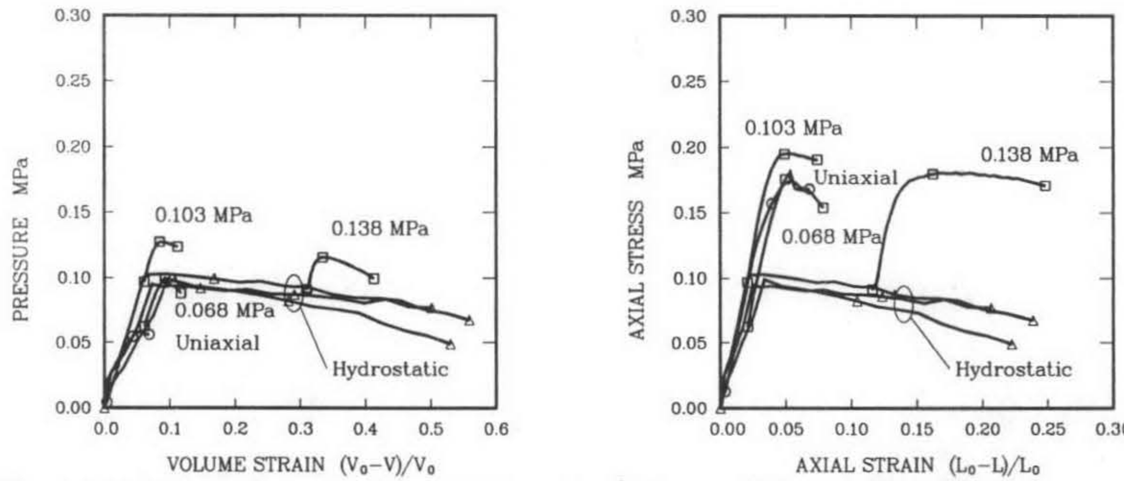


Figure 2: Skeleton Response for 0.032 gm/cm^3 Foam - Triaxial Specimens were Hydrostatically Loaded to the Labeled Pressure, then Axial Load Alone was Increased.

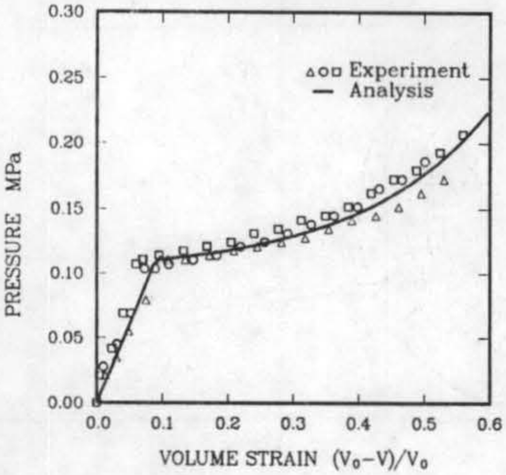
step function, γ is the volume strain or first invariant of the foam strain, and A, B, and C are material constants. B is the yield strength of the skeleton for purely hydrostatic loading, and C defines the skeleton's volumetric response after yielding for purely hydrostatic loading. Material constant A is equal to the difference between the axial yield strength for hydrostatic loading and the axial yield strength for deviatoric loading. The first term in Equation 5 is only active when the deformation is deviatoric. The principal skeleton stresses must be less than or equal to the yield function, g. If the principal skeleton stresses are less than g, the behavior is elastic. If the principal skeleton stresses are equal to g, the behavior may be plastic. Results from one hydrostatic test and one triaxial or uniaxial test are needed to define the material constants for this new model. Material constants for two different foams are given in Table 1.

Table 1: Mechanical Properties

Foam Density (gm/cm^3)	Young's Modulus (MPa)	A (MPa)	B (MPa)	C (MPa)	ϕ
0.032	3.2	0.066	0.107	0.079	0.035
0.080	20.8	0.339	0.419	-0.216	0.090

The next step in the development of the new constitutive model was its implementation in finite element computer codes. The model was incorporated in SANCHO (Stone et al., 1985), a quasistatic dynamic relaxation code, and in PRONTO (Taylor and Flanagan, 1986), a transient dynamics code. The implementation in both codes was straightforward.

The last step in the development of this new constitutive model was to verify that this model accurately represented the polyurethane foam behavior. To verify the model, a series of analyses was completed using the new constitutive model in SANCHO and PRONTO. This series of analyses was completed using an axisymmetric, one element model of a NMERI test sample. Boundary conditions



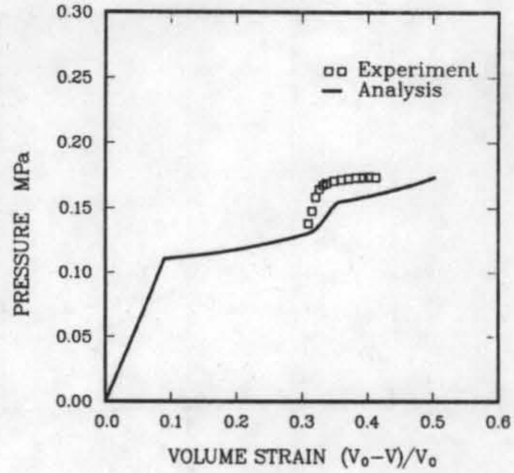


Figure 3: Comparison of Analytic and Experimental Results - Hydrostatic and Triaxial Tests on 0.032 gm/cm^3 Foam.

on the model were varied to represent the various NMERI tests. Experimental foam behavior and constitutive model behavior from hydrostatic and triaxial tests on 0.032 gm/cm^3 foam are shown in Figure 3. The new constitutive model accurately represented the foam behavior observed in the NMERI tests.

SOLUTION OF AN IMPACT PROBLEM USING THE NEW MODEL

The new foam constitutive model and PRONTO were used to analyze the impact of an infinitely long steel cylinder surrounded by a foam layer with a thin aluminum shell. This problem was chosen to demonstrate the capabilities of the model for handling complex stress states. Results from this analysis were compared with results obtained using a conventional deviatoric plasticity model and a cap model to demonstrate the effects of using the various models for the foam material. The two dimensional, plane strain finite element model shown in Figure 4 was used in these analyses. The cylinder was dropped onto a rigid surface at an initial velocity of 13.4 meters per second, and the resulting deformations and accelerations were computed. The foam layer was assumed to be 0.080 gm/cm^3 foam and was modeled with the three different constitutive models discussed above.

Displaced shapes of the finite element model at maximum crush-up are shown in Figure 5. Plots of displacement and acceleration of the steel cylinder as a function of time are shown in Figure 6. Peak acceleration predictions obtained using the new constitutive model are between predictions obtained using the conventional deviatoric plasticity model and the cap model. The conventional deviatoric plasticity model does not allow for any volumetric plasticity and is stiffer than the other two models. The cap model assumes the volumetric response is not dependent on deviatoric loading and is softer than the other two models.

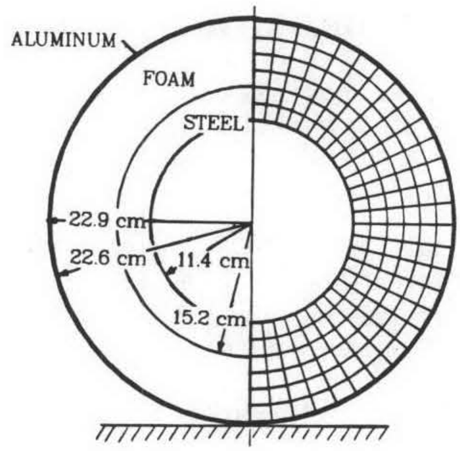
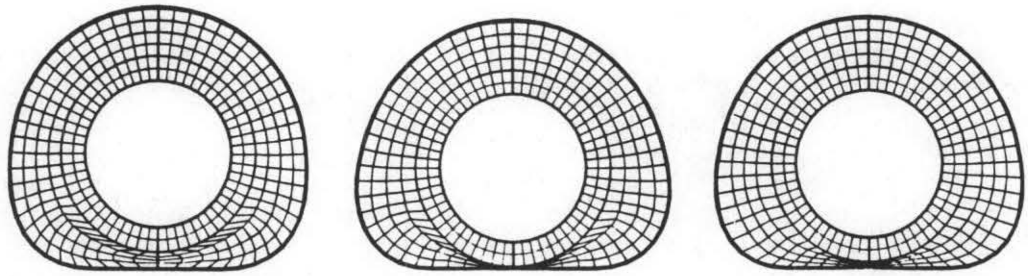


Figure 4: Finite Element Model.



a **b** **c**

a) conventional model **b) cap model** **c) new foam model**

Figure 5: Displaced Shape of Finite Element Model.

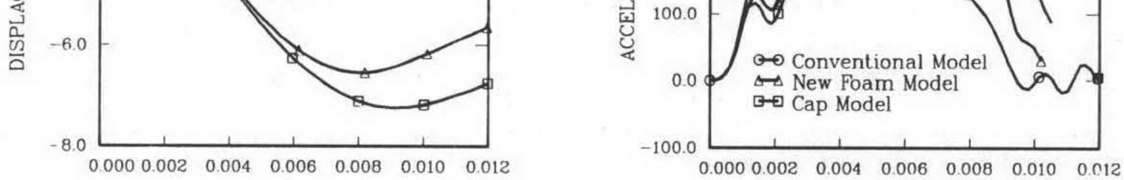


Figure 6: Displacement and Acceleration of Steel Cylinder.

CONCLUSIONS AND FUTURE WORK

The behavior of rigid, closed-cell, polyurethane foam was experimentally investigated. These experiments indicated that these foams undergo large plastic volume strains when subjected to sufficient load and that the deviatoric and volumetric behaviors for these foams are coupled.

A conventional deviatoric plasticity model and a cap model did not capture foam behaviors observed in the NMERI experiments. Thus, a new constitutive model for rigid, closed-cell, polyurethane foams was developed. This constitutive model was implemented in two finite element codes, SANCHO and PRONTO. A typical problem was analyzed using this new constitutive model and other constitutive models to demonstrate differences between the various models.

Because the experimental NMERI tests were all static tests, there was no way to determine if rate effects were important; therefore, no rate effects were included in the new constitutive model. In the future, dynamic tests should be completed to determine if rate effects are important. The new constitutive model could then be modified, if necessary, to include any important rate effects. Temperature effects were also not investigated in the NMERI tests. Temperature effects were included in the new constitutive model by assuming that the air behaves as an ideal gas and that temperature changes have no effect on the polymer skeleton. Experimental tests should be completed to investigate the effects of temperature variations on the skeleton response. Once the new constitutive model has been modified to include any important rate or temperature effects, it could then be used with confidence to analyze dynamic events. Future comparisons between experimental results and analyses with this constitutive model would further increase confidence in its accuracy.

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