Force and Moment Reconstruction for a Nuclear Transportation Cask Using Sum of Weighted Accelerations and Deconvolution Theory*

H.R. Yoshimura, V. Bateman, T.G. Carne, D.L. Gregory, S.W. Attaway, D.R. Bronowski

*Sandia National Laboratories**, Albuquerque, New Mexico, United States of America*

INTRODUCTION

A 9-m drop test was conducted of a 1/3-scale-model spent fuel cask onto an unyielding target. The structural response of the impact limiters and attachments was evaluated. A mass model of the cask body, with steel-sheathed redwood and balsa impact limiters, was tested in a 10-degree slapdown orientation. One end of the cask impacted the target before the other end, as shown in Figure 1, with higher deceleration forces resulting from the second impact. The information desired from this test is the deformation of the two impact limiters on either end of the cask as a function of the applied force. The content in this paper will only discuss a summary of the applied force calculations. Additional details about the force and moment reconstruction methods and analysis results (Bateman et al., 1989) and test and hardware (Yoshimura et al., 1989) are provided elsewhere.

Two new force reconstruction techniques were applied to the slapdown test data: the sum of weighted accelerations technique (SWAT) and deconvolution (DECON). The conventional method for determining the resultant force involves post-test digitally filtering an accelerometer measurement to find the rigid-body acceleration for the cask. The rigid-body acceleration is then multiplied by the cask mass to obtain an estimate of the applied force. The frequency content of this force is restricted to the cut-off frequency of the digital filter, typically about one-half of the lowest elastic mode of the cask. The conventional method also restricts the rise time of the force to the rise time of the digital filter. The new force reconstruction techniques demonstrate the potential for a better estimate of forces acting on the cask during the impact than the conventional method.

The new force reconstruction techniques use the cask structure as a generalized force transducer. With these techniques, the elastic vibration response of the cask is eliminated from the acceleration data. This elastic vibration response does not contribute to the rigid body translation or rotation of the cask. The main advantages of the force reconstruction techniques are the extension of the frequency bandwidth (due to the elimination of the elastic modal response in that bandwidth) and the preservation of the force rise time.

CASK STRUCTURAL DYNAMIC CHARACTERIZATION

To determine the data needed for the force reconstruction techniques, a modal test was performed. The cask was supported softly by elastic straps to simulate free boundary conditions. The cask was excited using an instrumented impact hammer. From the measured excitation and the resulting acceleration response data, frequency response functions (FRFs) were calculated, and modal

* This work performed at Sandia National Laboratories, Albuquerque, New Mexico, supported by the United States Department of Energy under Contract DE-AC04-76DP00789.

** A United States Department of Energy Facility parameters were estimated from the FRFs. The measured modal frequencies and their corresponding mode shapes provided information for the SWAT force reconstruction.

Figure 2 displays three mode shapes overlaid with the undeformed cask geometry. The seven rings shown on the undeformed geometry each represent an axial location where triaxial accelerations were measured at eight circumferential locations about the cask. Additional rings of accelerometers were mounted on each end cap. Mode shape coefficients were computed at each of these locations.

If the conventional technique was to be used to obtain the force for this cask geometry by filtering the acceleration data and multiplying the result by the mass, then the cut-off frequency for filtering acceleration data to eliminate elastic response would be ~500 Hz, or about half the first modal frequency. A frequency bandwidth of 1500 Hz may be obtained through force reconstruction techniques by eliminating the lowest five vibration modes from the data shown in Table 1, thus providing three times the frequency content available from the conventional technique of filtering acceleration data. If more accelerometers are added to capture higher frequency vibration modes, the frequency response could be extended even higher.

Table 1: A Comparison of Analytical Predicted and Experimentally Measured Elastic Modes for the Cask

Description	Harmonic Number (n)	Analytically Predicted (Hz)	Experimentally Measured (Hz)
First Ovaling Mode	2	981	972
First Bending Mode	1	978	992
First End-Cap Mode	0	1438	1076
Second End-Cap Mode	0	1869	1180
Second Ovaling Mode	2	1385	1340
Second Bending Mode	1	1487	1528
First Axial End-Caps Out-of-Phase Mode	0	2023	1788
Third Bending Mode	1	1915	1856
Third End-Cap Mode	0	2580	2036

Each of the two new force reconstruction techniques eliminates, by different methods, the elastic vibratory response of the cask from the acceleration data. SWAT uses the cask modal characteristics to calculate weighting factors (Bateman et al., 1989). Acceleration measurements from different points on the cask are multiplied by these weighting factors and summed to reconstruct the force. The DECON technique uses measured frequency response functions; these functions can be combined with the measured response to reconstruct the force (Bateman et al., 1989). Both techniques were demonstrated with laboratory tests prior to the slapdown event, and known force inputs were applied to the cask to calibrate it as a force transducer.

SUM OF WEIGHTED ACCELERATIONS TECHNIQUE (SWAT)

The SWAT approach determines the external force applied to a structure with free boundary conditions from a sum of weighted acceleration measurements (Gregory et al., 1986; Priddy et al., 1988; Priddy et al., 1989). Two sets of weighting factors are determined through modal analysis: one set, described in units of mass, is used to reconstruct the resultant force vector and the other set, described in units of first-moment-of-mass, is used to reconstruct the resultant moment vector. These factors eliminate the elastic response of the cask from the acceleration data. The number of vibration modes that can be eliminated from the force reconstruction is limited by the number of accelerometers. More locations will allow more vibration modes to be eliminated. The number of accelerometers required for force reconstruction in a particular frequency bandwidth is determined by the number of vibration modes of the structure in that bandwidth.

The weighting factors multiply the measured acceleration signals to reconstruct the applied force and moment. A set of weights can be calculated for each vector component of the force and moment. In this cask study, rings of symmetrically located accelerometers were used to eliminate certain circumferential harmonic modes, thus decreasing the modes in a particular bandwidth and increasing the number of modes cancelled. As a result, the weighting equations for the axisymmetric cask were simplified to those for a rod or beam.

DECONVOLUTION (DECON) TECHNIQUE

The DECON technique combines measured accelerations with impulse response functions to infer the external force applied to a structure with free boundary conditions. The method requires that the point and line of action of application of the forces be known. Forces applied at other points contaminate the DECON method and cannot be separated from the desired forces without reformulation of the problem. Theoretically, the number of accelerometers required is equal to the number of forces to be reconstructed, but in application the technique is more successful when there are more accelerometers than forces (Hillary and Ewins, 1984). The DECON technique can be implemented with either accelerometers or strain gages and in situations with a limited number of instrumentation channels (Bateman, 1989).

The DECON method separates the influence of vibration modes from the external forcing function applied to the cask body structure. The reconstruction is computed using a division in the frequency domain which is equivalent to deconvolution in the time domain. This concept may be represented mathematically as the Fourier transform of the input equalling the Fourier transform of the response vectors multiplied by the inverse of the matrix of the structural frequency response functions (FRFs).

The structural FRF, which is measured in the laboratory, is the frequency-domain representation of the time-domain impulse response function. The Fourier transform of the forcing function during the drop test may be determined by dividing the Fourier transform of the output obtained from acceleration data by the structural FRF obtained from laboratory measurements.

This technique implicitly assumed that the lateral forces were applied at the two cask ends for the slap-down impact. Responses were measured at four locations. By averaging the responses at the same axial location, but 180 degrees apart along the circumference, the effect of the circumferential harmonic modes was eliminated (as explained previously for SWAT).

IMPLEMENTATION AND CALIBRATION OF RECONSTRUCTION TECHNIQUES

Both techniques, SWAT and DECON, were implemented in algorithms using the interactive GENRAD 2515 data acquisition system. The reconstructed forces were computed and displayed as soon as the accelerometer responses were acquired. With both force reconstruction algorithms, the cask acts like a force transducer which can measure the externally applied force. As with any force transducer, the cask must first be calibrated to verify the assumed relationships. The cask was suspended in the same manner as for the modal test. Instrumented impact hammers were calibrated and used to apply one or more forces. The cask accelerometer responses were measured simultaneously and used to reconstruct the input forces. The input forces were also measured directly using a force transducer on an impact hammer. The measured forces were directly compared with the reconstructed forces (see Figure 3), with very good results.

Calibration tests were also used to evaluate the sensitivity of the SWAT algorithm to an axial input. With an axial force application, the reconstructed lateral force yields a result close to zero, confirming the insensitivity of the algorithm to forces other than the desired lateral force. A multi-input test showed that the algorithm can reconstruct the force input even when there are multiple applications at different points on the structure. Similar experiments were performed to evaluate and calibrate the moment reconstruction algorithm, also producing excellent correlations.

For the DECON technique, a two-by-two matrix of FRFs was inverted and stored on the GENRAD 2515. Since the DECON method assumes separate input forces at either cask end, two separate force-time histories are computed (in contrast to the composite force computed with the SWAT algorithm). A comparison of a measured and reconstructed force for a single impulsive force at one end showed excellent agreement.

In another test, two separate pulses were applied at different ends of the cask about thirty milliseconds apart. The DECON reconstruction for the two inputs compared well with the known inputs. Further, the accuracy of DECON appears to be unaffected by the occurrence of the second force. A direct comparison between the reconstructed and measured forces again shows excellent agreement.

These calibration experiments show that the algorithms produce accurate applied force reconstructions during the force pulse. Both techniques show some difficulty in reconstructing the zero-magnitude signal after the force application is over. The magnitude of the error, however, was small compared with the magnitude of the force input.

SLAPDOWN TEST RESULTS

For the slapdown test, eight accelerometers were used for the force reconstruction. Two accelerometers at each of four axial locations were mounted 180 degrees apart and aligned circumferentially with the cask so that they pointed in the vertical (impact) direction. All eight acceleration responses were used for the SWAT algorithm, but only four were used in the DECON algorithm. In both cases, the two acceleration responses at an axial location were summed to eliminate circumferential harmonic modes. Since the cask was dropped with a 10° inclination (12° measured) to the horizontal, an impact first on one end and then on the other end produced two separate impact forces. It was uncertain to what extent the forces would overlap in time. The DECON algorithm computes these impact forces as separate functions. In contrast, the SWAT algorithm computes the resultant force and the moment about the center-of-mass. The calculated forces and moments are those applied to the cask body by the impact limiters and are not the total force applied to the entire system including the limiters, i.e., the force applied to the limiters by the ground.

Using the SWAT algorithm, the impact force as shown in Figure 4a lasted approximately thirty-five milliseconds with a peak value of about 500,000 pounds. The two force pulses that result from the slapdown impact are clearly distinguished; the second pulse has a slightly higher peak value as expected (Sjaardema and Wellman, 1988). As a validity check on the reconstruction, the force was integrated and the calculated impulse was compared to the change in momentum at impact. With the assumption that the cask is at rest at the end of the event, this calculated impulse was within one percent of the theoretical free-fall momentum.

The corresponding moment about the center-of-mass, calculated using SWAT, reflects the two lobed character of the applied force as shown in Figure 4b. The moment is at first positive, and then changes sign as the cask slaps down.

The applied moment was integrated and divided by the cask moment of inertia to calculate the angular velocity during the impact (Figure 5). This calculation serves as another check on the reconstruction because the angular velocity was zero at the beginning and at the end of the event. The calculated angular velocity shows only a very slight deviation from zero at the end of the record. These data were also compared to angular velocity measurements from high-speed photography. The two results were virtually identical.

The external forces were also reconstructed using DECON, assuming two impact forces perpendicular to the cask axis were applied at either cask end as shown in Figure 6. The same character is evident in these forces as those reconstructed using SWAT: a large positive force at the primary impact followed by another large force at the second impact end. The two reconstructions also show interactions between the cask and the limiters outside the respective impact durations.

A comparison of the force reconstruction from SWAT to that from DECON is made by adding the two forces from DECON to obtain the total external force. The results are very similar as shown in Figure 7. The primary impact shows extremely close agreement between the two reconstructions.

For the second impact, DECON produces slightly larger values (about 68,000 lb) but the shape of the force is quite similar including all the high-frequency spikes. For comparison with the conventional method, a force computed from acceleration data filtered at 500 Hz is also shown in Figure 7.

CONCLUSIONS

Two force reconstruction techniques, SWAT and DECON, were used to evaluate the structural response of impact limiters on a 1/3-scale-model cask during a 10-degree slapdown impact test. Prior to conducting the drop test, laboratory experiments were performed to calibrate both reconstruction techniques. Measured and reconstructed forces from single- and multiple-instrumented hammer impacts on various parts of the cask body demonstrated excellent correlation under laboratory conditions.

In the field, both reconstruction techniques provided very similar results for the resultant applied force to the cask body. These reconstructions were significant improvements of the slapdown force estimate due to the extended bandwidth of 1500 Hz. This bandwidth is about three times the bandwidth available from force estimates with digitally filtered data using conventional methods to determine cut-off frequency. The conventional method has yielded the two lobed nature of the applied force, but the rise-time and fall-time of the impact force has been significantly overestimated. For example, DECON shows a rise-time of 2.8 ms while the conventional method yields 5.7 ms. The error in force rise-time could affect the design of the impact limiters. As shown in the results, the resolution of the forces by SWAT and DECON was provided in greater detail over conventional filtering. The 500 Hz filtered data smeared the details of the forces imposed on the cask body during crush up of the limiters. The higher resolution provided by the reconstruction techniques can assist designers to better understand the deformation process during crush.

In addition, the SWAT yielded a reconstructed moment for the slapdown test. This information was previously unavailable through conventional reduction of cask drop test accelerometer data. The angular velocity calculated with SWAT was verified with photometric measurements. In summary, the two techniques provide significant improvements over conventional data reduction methods for evaluating the structural response of cask impact limiting systems.

REFERENCES

Bateman, V. I. "Force Reconstruction for the G-M118 Field Test of the Strategic Earth Penetrator Weapon," SAND88-3046·UC-32, National Technical Information Service, Springfield, VA 22162 (1989).

Bateman, V. I., Carne, T. G., Gregory, D. L., Attaway, S. W., and Yoshimura, H. R. "Force Reconstruction for the Slapdown Test of a Nuclear Transportation Cask," SAND88-3045, Sandia National Laboratories, Albuquerque, NM (1989).

Gregory, D. L., Priddy, T. G., and Smallwood, D. O. "Experimental Determination of the Dynamic Forces Acting on Non-Rigid Bodies," SAE Technical Paper Series, Paper No. 861791, Aerospace Technology Conference and Exposition, Long Beach, CA (1986).

Hillary, B., and Ewins, D. J. "The Use of Strain Gages in Force Determination and Frequency Response Function Measurements," Proceedings of the 1984 Internal Modal Analysis Conference, pp. 627-634, Orlando, FL (1984).

Priddy, T. G., Gregory, D. L., and Coleman, R. G. "Strategic Placement of Accelerometers to Measure Forces by the Sum of Weighted Accelerations," SAND87-2567·UC-38, National Technical Information Service, Springfield, VA 22162 (1988).

Priddy, T. G., Gregory, D. L., and Coleman, R. G. "Measurement of Time-Dependent External Moments by the Sum of Weighted Accelerations," SAND88-3081·UC-38, National Technical Information Service, Springfield, VA 22162 (1989).

Sjaardema, G. D., and Wellman, G. W., "Numerical and Analytical Methods for Approximating the Eccentric Impact Response (Slapdown) of Deformable Bodies," SAND88-0616-UC-71, National Technical Information Service, Springfield, VA 22162 (1988).
Yoshimura, H. R., Attaway, S.W., Bronowski, D. R., Uncapher, W. L., Huerta M., Abbott, D. G., "1/3-Scale Model Testing Program," PATRAM'89, Washington, D.C. (1989)

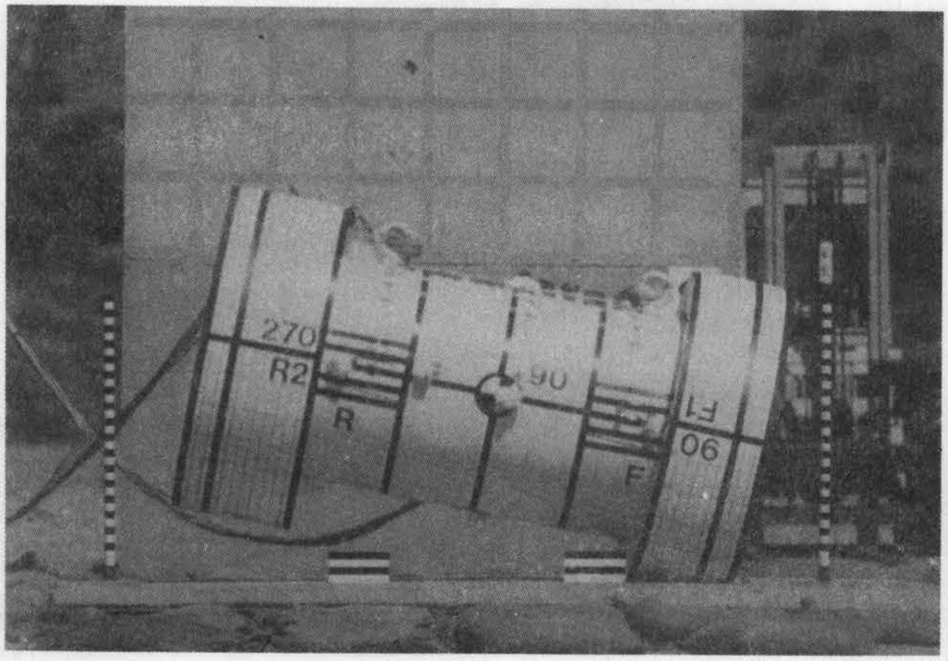


Figure 1. Slapdown Impact Test for a Nuclear Transportation Cask

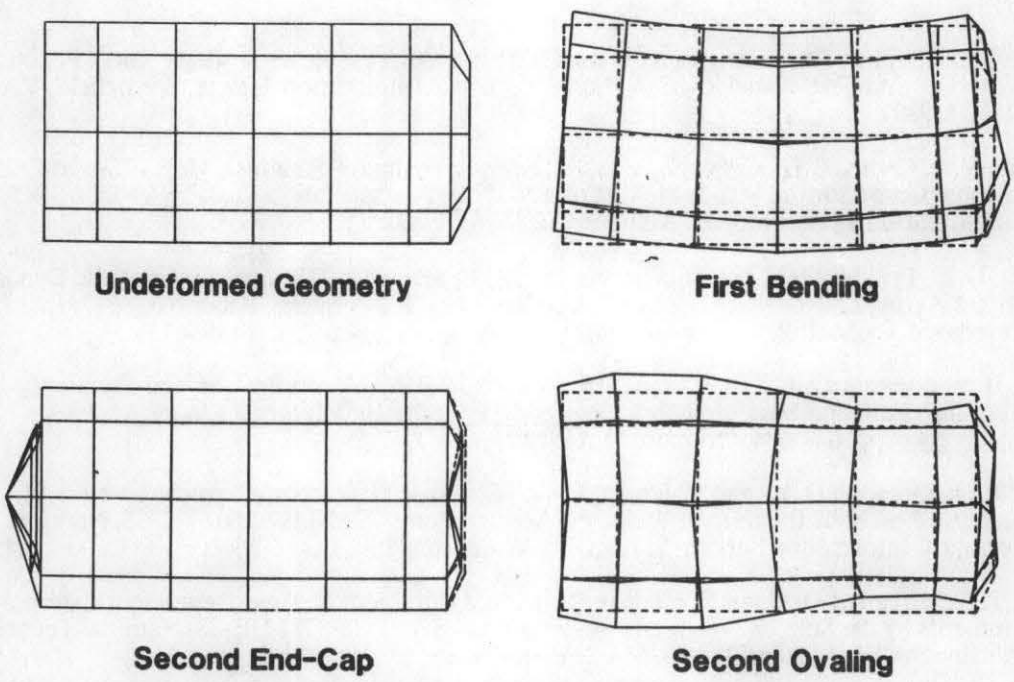
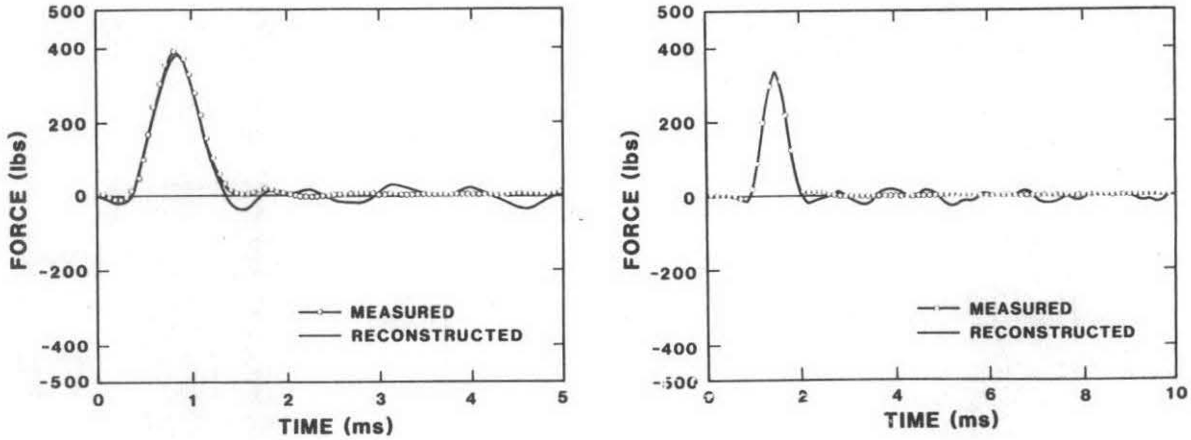
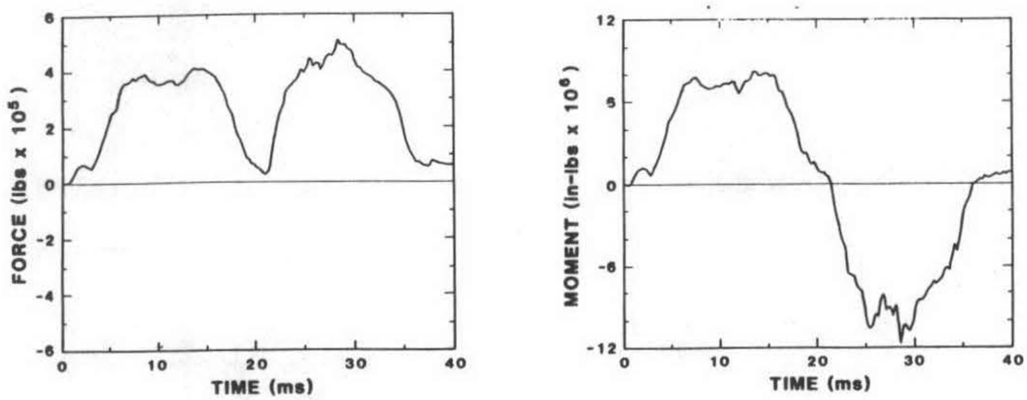


Figure 2. Typical Experimental Mode Shapes for the Cask Body



a) b)

Figure 3. Comparison of Measured Force and Reconstructed Forces by a) SWAT and b) DECON in the Laboratory



a) b)

Figure 4. Reconstructed a) Force and b) Moment from the Slapdown Test Using SWAT

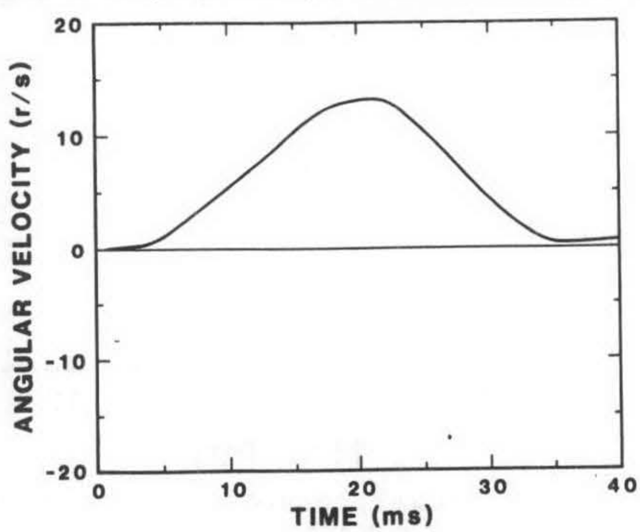


Figure 5. Angular Velocity of Cask During Impact as Evaluated by SWAT

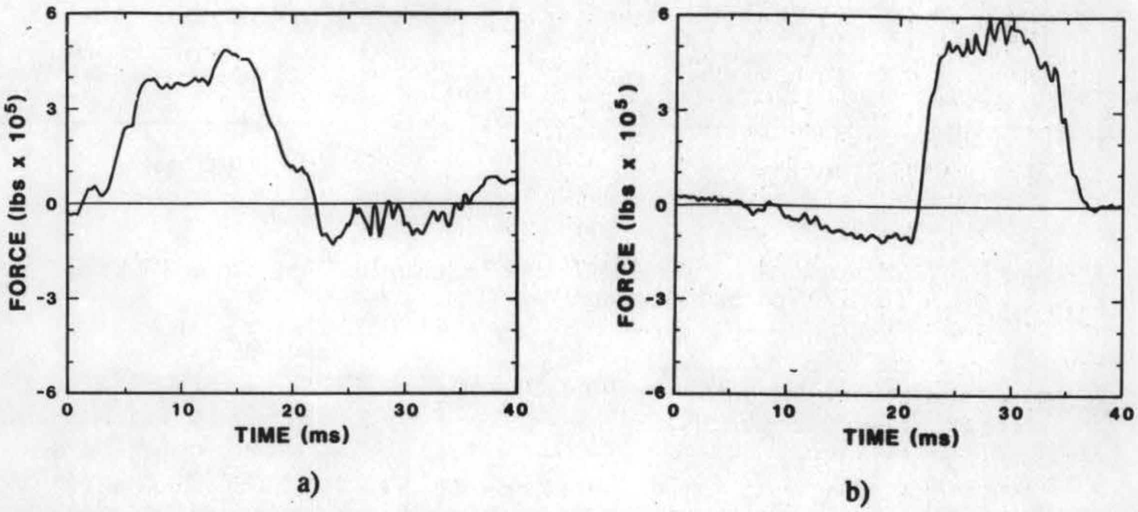


Figure 6. Force Reconstruction for a) Primary Impact and b) Secondary Impact Using DECON

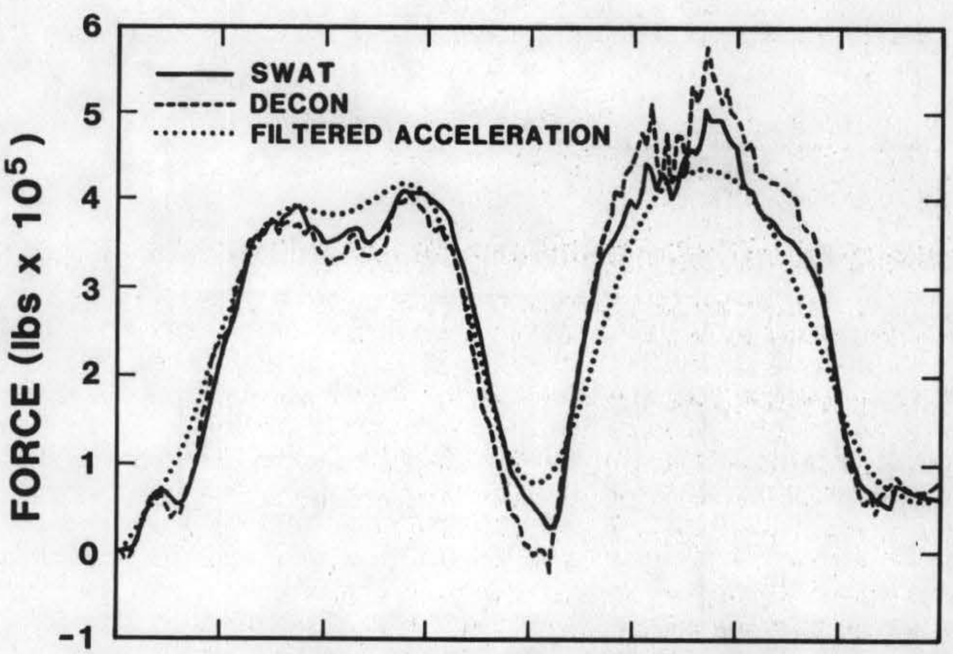


Figure 7. Comparison of Reconstructed Force for SWAT and DECON with Force Estimated by the Conventional Filtering Technique