IAEA-SM-286/128P

PHASE CHANGE MODELS FOR CASK ANALYSIS

W.H. LAKE
United States Nuclear Regulatory Commission,
Washington, D.C.,
United States of America

Abstract

PHASE CHANGE MODELS FOR CASK ANALYSIS.

The applicability of the pure conduction model is considered for lead-shielded casks in which melting occurs. By modifying the classical pure conduction melt model, the inadequacy of this model for melting in lead shielded casks is demonstrated.

1. INTRODUCTION

The purpose of this paper is to consider the applicability of the classical formulation of the phase change problem for the thermal analysis of lead shielded packaging used to transport radioactive material. The classical model assumes that the conduction mode of heat transfer prevails in the lead shield for both the solid and liquid phases [1]. Although the exact classical solution is not typically used by thermal analysts, the classical model appears in numerical form in at least one important and frequently used heat transfer computer program [2]; it is probably used for other heat transfer programs.

On the basis of a fire test reported by Wachtell and Langhaar [3], it is evident that natural convection heat transfer should be considered if molten lead is present in a cask. Furthermore, analytic investigation of phase change using a conduction-convection melt model suggests a number of difficulties associated with the classical pure conduction model [4]. The difficulties associated with the pure conduction model include possible under prediction of the progress of the melt front, and the erroneous prediction of a uniform melt front profile along the height of the melting region.

When analysis is used to demonstrate compliance with international [5], United States [6], or other transportation regulations with regard to shielding adequacy, the extent of lead melt in a cask may be important. An accurate estimate of the shielding configuration is needed to assess the radiation protection provided by the packaging.

2. THEORETICAL BACKGROUND

2.1 Physical Description

The physical phenomena of interest are heat transfer by conduction and natural convection and phase change. Conduction heat transfer occurs in solids, liquids, or gases. Natural convection heat transfer occurs in liquids or gases. For natural convection, the driving force is buoyancy of a heated fluid. The phase changes of interest are between the solid and liquid phases.

Consider a large lead system encased in a thin highly conductive shell and initially at constant temperature below melt. The surface temperature is raised at a uniform rate to a temperature above melt.

Initially, a narrow melt region is formed with little mixing of the liquid lead; conduction heat transfer dominates in the liquid region. As the melt front progresses, the liquid region becomes larger and the buoyancy induced mixing increases, resulting in enhanced heat transfer by natural convection and increased progress of the melt front. The mixing is initially laminar; as the melt region becomes larger and the temperature differences greater the mixing becomes turbulent. For laminar convection in the melt region, the heat transfer is found to vary with the height for a vertical system [7] [8]. For turbulent heat transfer height dependence is not expected [8].

2.2 Classical Mathematical Description (Pure Conduction)

The pure conduction model considered is the one-dimensional (x) melting of a semi-infinite region with a constant temperature surface condition. The melt front location, S, and the temperature distributions, T_l and T_s in the liquid and solid regions are found in terms of time, t; surface temperature, T_B; melt temperature, T_m; thermal conductivity, k_i; thermal diffusivity, α_i; and density, ρ. The subscript, i, is either s or l for solid or liquid. The solution [1] is presented in equations (1), (2) and (3), assuming an initial temperature, T_o.

$$S = 2\lambda\sqrt{\alpha_l t} \tag{1}$$

$$T_l = T_B - \frac{(T_B - T_m)}{\text{erf}\,\lambda}\ \text{erf}\ \left(x / 2\sqrt{\alpha_l\, t} \right) \tag{2}$$

$$T_s = T_o + \frac{(T_m - T_o)}{\text{erfc}\ (\lambda\sqrt{\alpha_l/\alpha_s})}\ \text{erfc}\ (x/2\sqrt{\alpha_s t}) \tag{3}$$

where the heat of fusion, h_f, and the specific heat, c_l, are used to find λ, the root of equation (4)

$$\frac{\lambda e^{-\lambda^2}}{\operatorname{erf}\lambda} - \frac{k_s\sqrt{\alpha_l}\,(T_m - T_o)e^{-\lambda^2(\alpha_l/\alpha_s)}}{k_l\sqrt{\alpha_s}\,(T_B - T_m)\operatorname{erfc}(\lambda\sqrt{\alpha_l/\alpha_s})} = \frac{h_f\lambda\sqrt{\pi}}{C_l(T_B - T_m)} \qquad (4)$$

The problem is simplified by specifying an initial temperature equal to the melt temperature ($T_o = T_m$). The solid temperature, T_s, is then constant and equal to the melt temperature, T_m. Equations (1) and (2) are unchanged, but the root equation, (4), is simplified. The simplified and rearranged root equation is:

$$\lambda e^{\lambda^2} \operatorname{erf}(\lambda) = c_l\,(T_B - T_m)/(h_f\sqrt{\pi}) \qquad (5)$$

2.3 Natural Convection

Natural convection is considered in terms of the dimensionless groups: Nusselt number, Nu, which is the ratio of convection to conduction heat transfer; Grashof number, Gr, which is the ratio of buoyant and viscous drag forces; Prandtl number, Pr, which is the ratio of momentum and thermal diffusivities and Rayleigh number, Ra, which is the product of Gr and Pr.

The Nusselt number correlations used in the paper for vertical plates with laminar or turbulent heat transfer are [8]:

$$\text{Laminar:}\quad Nu_L = 0.3\, Ra_y^{0.25},\quad Ra < 10^6 \qquad (6)$$

$$\text{Turbulent:}\quad Nu_T = 0.028\, Ra_x^{0.355},\quad 4\times 10^4 < Ra < 10^8 \qquad (7)$$

where y = vertical height, x = distance between vertical surfaces.

The Rayleigh number is given in terms of the volume expansion coefficient, β; the temperature difference, ΔT; density, ρ; thermal diffusivity, α viscosity, μ; gravity, g and characteristic length, L

$$Ra = (g\beta\rho\,\Delta T\, L^3)\,/\,(\alpha\mu) \qquad (8)$$

TABLE I. PROPERTY VALUES

Property	Symbol	Phase*	Value
density	ρ	s,l	10560 kg/m^3
specific heat	c	s,l	0.161 kJ/kg-°C
conductivity	k_s	s	34.6 W/m-°C
	k_l	l	16.3 W/m-°C
thermal	α_s	s	20.2×10^{-6} m^2/s
diffusivity	α_l	l	9.5×10^{-6} m^2/s
melt temperature	T_m	-	327°C
heat of fusion	h_f	-	23.34 kJ/kg
viscosity	μ	l	2.41×10^{-3} $N\text{-}s/m^2$
volume expansion	β	l	1.14×10^{-4}/°C
gravity constant	g	-	9.8 m/s^2
Prandtl number	Pr	l	0.0239
Rayleigh group	Ra/L^3 T	l	$5.13 \times 10^8/(°C\ m^3)$

*s = solid, l = liquid

3. ANALYSIS

3.1 Properties

Constant properties are used for analysis. The property values are taken at 355°C which is just above the melt temperature of lead (327°C); however, conductivity of solid lead is taken at the melt temperature. Table I gives the values used.

3.2 Analytic Models

Three models are considered. The first is the classical one-dimensional model used to find the time dependent melt front location. The remaining two use effective thermal diffusivity to account for natural convection in the melt to predict the time dependent melt front location.

3.2.1 Model 1

This is the classical one-dimension model [1] of a semi-infinite lead system, initially at the melt temperature

TABLE II. SELECTED SOLUTIONS FOR MODEL 1

$\Delta T=T_B-T_m$ (°C)	λ	Melt Front S (m)			
		t=900s	t=1800s	t=2700s	t=3600s
1	0.05879	0.011	0.016	0.019	0.022
10	0.18368	0.034	0.048	0.059	0.068
50	0.39420	0.073	0.104	0.127	0.147
100	0.53482	0.099	0.141	0.172	0.199
200	0.70140	0.130	0.185	0.226	0.261

($T_B=T_m$=327°C). The surface temperature is raised to T_B at t = 0 and held constant. Substitution of the property values from Table I into equation (5) yields an equation for the root, λ, in terms of the temperature difference $\Delta T=(T_B-T_m)$

$$\lambda e^{\lambda^2} \text{erf}(\lambda) = (T_B - T_m)/257 \qquad (9)$$

Substituting the thermal diffusivity, $\alpha_1 = 9.5\times10^{-6}\,m^2/s$, into equation (1) yields

$$S = 6.2\times10^{-3}\lambda\sqrt{t} \qquad (10)$$

Solutions of equations (9) and (10) for selected values of ΔT and t are given in Table II.

3.2.2 Model 2

The classical one-dimensional model is used with the same specifications as Model 1; however, an effective thermal diffusivity is used and the surface temperature is held constant. The effective thermal diffusivity, α_{eff}, is defined as the product of Nu and α_1. The Nusselt number correlation for turbulent natural convection is used. Based on fire test results for a cask [3], a surface temperature, T_B, of 527°C is assumed

$$\alpha_{eff} = Nu_T\alpha_1 \qquad (11)$$

using the Table I data, $\Delta T = T_B - T_m$ = 200°C and equations (7) and (8)

$$Ra = 1.03 \times 10^{11} x^3 \qquad (12)$$

$$N_{u,T} = 2.27 \times 10^2 x^{1.065} \qquad (13)$$

TABLE III. SELECTED SOLUTIONS FOR MODEL 2

x(m)	Ra	Nu	Melt Front S (m) t=25s	t=30s	t=40s	t=50s	t=60s
0.05	1.29×10^7	9.3	0.066	0.073	0.084	0.094	0.103
0.10	1.03×10^8	19.5	0.096	0.105	0.121	0.136	0.149
0.15	3.48×10^8	30.1	0.120	0.132	0.152	0.170	0.186

For $\Delta T = 200°C$, $\lambda = 0.7014$ and the melt front is equation

$$S = 4.35 \times 10^{-3} \sqrt{Nu_T t} \tag{14}$$

Solutions of equations (12), (13), and (14) for selected values of x and t are given in Table III. Because of the rapid progress of the melt front for this model, values of $t \leq 60s$ are considered.

3.2.3 Model 3

The third model is a two-dimensional slab, 0.5 m high by 0.1 m thick, insulated on three sides and initially at the melt temperature (327°C). The effective thermal diffusivity is used again, but the surface temperature, T_B, is 328°C ($\Delta T = 1°C$). For this case, laminar natural convection heat transfer is assumed and Nu depends on height

$$\alpha_{eff} = Nu_L \alpha_1 \tag{15}$$

using Table I data, $\Delta T = T_B - T_m = 1°C$, and equations (6) and (8)

$$Ra = 5.13 \times 10^8 y^3 \tag{16}$$

$$Nu_L = 45.15 \quad y^{0.75} \tag{17}$$

For $\Delta T = 1°C$, $\lambda = 0.05879$, and the melt front equation is

$$S = 3.645 \times 10^{-4} \sqrt{Nu_L t} \tag{18}$$

Solutions of equations (16), (17), and (18) are given in Table IV for y's of 0.1m, 0.2m, 0.3m, 0.4m and 0.5m, and for t of 1/4 h (900s) and 1/2 h (1800s).

TABLE IV. SOLUTIONS FOR MODEL 3

y(m)	Ra	Nu	Melt Front S (m) t=1/4 h (900s)	t=1/2 h (1800s)
0.1	5.13×10^5	8.03	0.031	0.044
0.2	4.10×10^6	13.5	0.040	0.057
0.3	1.39×10^7	18.3	0.047	0.066
0.4	3.28×10^6	22.7	0.052	0.073
0.5	6.41×10^6	26.8	0.056	0.080

3.3 Discussion of Analytic Results

The results given for Models 1 and 2 in Tables II and III show that the effective thermal diffusivities assumed result in much more rapid progress of the melt front than predicted by the pure conduction model. From Table II (pure conduction model), we find that it takes between 900s (1/4 h) and 1800s (1/2 h) to melt 0.15m when ΔT is 200°C. From Table III, the same melt is achieved in about 40s using the effective thermal diffusivity model.

For the laminar natural convection case (Model 3, Table IV) a comparison with the ΔT = 1°C case for pure conduction (Model 1, Table II) is made. Again, more melt is observed for Model 3 which uses an effective thermal diffusivity to account for convection. Also, a non-uniform melt profile is observed.

Before closing the discussion of the models considered, a cautionary note is needed. The adjusted pure conduction models used (Models 2 and 3) are intended only to show the potential deficiency of using a pure conduction model for lead systems where melting occurs. The method is not intended to predict the melt front location, nor is it intended to predict temperatures in the melt region.

4. CONCLUSION

The pure conduction model for lead melt in shipping casks does not accurately predict lead shield behavior under the fire accident tests specified in regulations [5] [6]. Although the method of using effective thermal diffusivity may prove useful it should not be used indiscriminately; furthermore, test or experimental verification is necessary to justify such a model for cask analysis. To develop a reliable approach to analysis of lead melting and solidification in casks, analytic and numerical models which adequately account for convection must be developed and verified through experiments and tests.

REFERENCES

[1] Carslaw, H. S., Jaeger, J. C., Conduction of Heat in Solids, Second ed., Oxford University Press, London, 1959.
[2] Elrod, D. C., et al, NUREG/CR-200, V2, Section F10, 1981.
[3] Wachtell, G. P., Langhaar, J. W., Nuclear Engineering and Design, 8, pp 273-288, 1968.
[4] Sparrow, E. M., et al, ASME, J of Heat Transfer, 99, pp 520-526, Nov 1977.
[5] IAEA, Safety Series No. 6, 1985.
[6] U.S., 10 CFR Part 71, Revised Jan. 1, 1985.
[7] Szekely, J. and Chhabra, P.S., Metallurgical Transactions, 1, May 1970, pp 1195-1203.
[8] Tang, Y. S., Coffield, R. D., and Markley, R. A., Thermal Analysis of Liquid-Metal Fast Breeder Reactors, ANS, 1978.